1D single-site and laterally constrained inversion of multifrequency and multicomponent ground-based electromagnetic induction data — Application to the investigation of a near-surface clayey overburden

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ABSTRACT

Electromagnetic induction (EMI) methods are widely used to determine the distribution of the electrical conductivity and are well adapted to the delimitation of aquifers and clayey layers because the electromagnetic field is strongly perturbed by conductive media. The multicomponent EMI device that was used allowed the three components of the secondary magnetic field (the radial H_r , the tangential H_{ϕ} , and the vertical H_z) to be measured at 10 frequencies ranging from 110 to 56 kHz in one single sounding with offsets ranging from 20 to 400 m. In a continuing endeavor to improve the reliability with which the thickness and conductivity are inverted, we focused our research on the use of components other than the vertical magnetic field Hz. Because a separate sensitivity analysis of H_r and H_z suggests that H_r is more sensitive to variations in the thickness of a near-surface conductive layer, we developed an inversion tool able to make single-sounding and laterally constrained 1D interpretation of both components jointly, associated with an adapted random search algorithm for single-sounding processing for which almost no a priori information is available. Considering the complementarity of H_r and H_z components, inversion tests of clean and noisy synthetic data showed an improvement in the definition of the thickness of a near-surface conductive layer. This inversion code was applied to the karst site of the basin of Fontaine-Sous-Préaux, near Rouen (northwest of France). Comparison with an electrical resistivity tomography tends to confirm the reliability of the interpretation from the EMI data with the developed inversion tool.

INTRODUCTION

Electromagnetic induction (EMI) sensors are suited for the detection of conductive targets and have been widely used to image geologic and hydrogeologic horizons. To implement EMI sounding, intercoil distance and frequency parameters need to be defined (Huang and Fraser, 1996; Sambuelli et al., 2007; Martinelli and Duplaá, 2008; Brosten et al., 2011) The desired depth of investigation is primarly used to define the ranges of these parameters. An important environmental issue consists in the evaluation of the thickness of a clayey overburden, particularly when considered as a protection against the pollution of karstic aquifers. In such a case, the interface defined by the resistivity contrast between the conductive overburden and the resistive component of the karst (the upper unsaturated part) is the principle target to be investigated. In the present study, we use a multicomponent, multifrequency EMI device, the PROMIS[®] (Iris instruments), which allows the user to simultaneously record induction data at 10 frequencies doubling from 110 to 56 kHz for all three components of the magnetic field $(H_r, \text{ along the axis of the source-receiver line, } H_{\phi}$, perpendicular to the source-receiver line, and H_z , the vertical component). In the present study, the transmitter orientation is set as a vertical magnetic dipole (VMD). Constant transmitter-receiver offset of 20 m and approximate heights of 1 m are used, unless otherwise noted.

When considering several frequencies and/or offsets, it is possible to estimate the variation of the resistivity as a function of the depth. To obtain these curves, several approximations could be used, such as the one presented by Fraser (1978), Sengpiel and Siemon (2000), and Siemon (2001), whose principal advantage is to avoid dealing with optimization algorithms and equivalences

Manuscript received by the Editor 26 September 2011; revised manuscript received 17 February 2012; published online 12 July 2012; corrected version published online 25 July 2012.

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inherent to the inverse problem. In that case, apparent resistivities are displayed versus centroid depth values. However, this approximation can only be used if the coil spacing is sufficiently small compared to the height above the ground, and thus suited essentially for airborne electromagnetic (AEM) methods.

It is not possible in the context that is presented to derive a similar pseudoresistivity curve by the use of the low-induction number (LIN) approximation, which relates the apparent resistivity value to the out-of-phase part of the vertical magnetic component H_z . Actually, the PROMIS configuration does not verify this approximation because of a large offset compared to the wavelength of the highest frequencies. For the same LIN approximation, Pérez-Flores et al. (2001) present linear approximation for fast 2D imaging, while, in the present study, we must deal with a nonlinear inversion problem.

Farquharson et al. (2003) present the application of a 1D inversion code to frequency AEM data for areas with high resistivities and high mineralization. The consideration of magnetic susceptibility for such mineral exploration is required as large values can be encountered, which is not really required when values of magnetic susceptibility are several decades smaller such as in sedimental environments, which are the focus of the present study. Moreover, the effect of the magnetic susceptibility decreases as long as the LIN approximation is less and less verified. In this case, the conductivity effect overwhelms all magnetic effects, both in the in-phase and the out-of-phase parts. In the same range of frequencies and offsets as the PROMIS device, Sasaki and Meju (2006) present the application of a 2.5D inversion code to get resistivity sections from a ground-based EMI device (with $H_z - H_z$ configuration only) in very complex fractured zones where multidimensional inversion is necessary to retrieve the correct geometry of the ground. Three-dimensional inversion codes for electromagnetic induction data also exist (Newman and Alumbaugh, 1997; Newman and Commer, 2005; Abubakar et al., 2006), but their computational cost and optimization efforts (Sasaki, 2001) are not justified in our study because the subsurface is almost 1D.

The efficiency of joint inversion procedures of multicomponent ground-based EMI data has been fully demonstrated in well logging applications (Kriegshäuser et al., 2000; Tompkins et al., 2004) where multicomponent data provide information about the electrical anisotropy. With a VMD as source, three different polarization configurations are possible: $H_z - H_r$ (perpendicular emitter-receiver or PERzx), $H_z - H_{\phi}$ (PERzy) and $H_z - H_z$ (horizontal co-planar or HCP). The $H_z - H_{\phi}$ configuration gives no response for a 1D layered ground and can only be used for multidimensional inversion.

For near-surface exploration with the transient AEM method, Auken et al. (2006, 2007) show an improvement of the definition of the upper part of the ground with the addition of the radial H_r component ($H_z - H_r$ configuration), especially for relatively conductive ground with a resistivity below 100 Ω m. Still, for AEM methods, Tølbøll and Christensen (2007) show that the $H_z H_r$ configuration has a similar lateral resolution as the $H_r - H_r$ or vertical co-axial (VCA) configuration, which is one of the most widespread configurations with $H_z - H_z$ and $H_{\phi} - H_{\phi}$ among ground-based EMI devices. It also gives more near-surface information compared to the horizontal loop configuration $H_z - H_z$. In the context of archeological prospection with ground-based EMI devices, Tabbagh (1986) shows that the $H_z - H_r$ configuration has the strongest response to a local 3D heterogeneity and a better depth of investigation than the $H_r - H_r$ configuration. Recently, McKenna and McKenna (2010) have shown the successful use of a multicomponent EMI device for imaging buried infrastructure. All these studies on the advantages of using the radial H_r component have lead us to consider its potential application in a joint inversion with the vertical H_r component.

In the present paper, a newly developed inversion algorithm dedicated to the interpretation of multicomponent, multifrequency, and ground-based EMI device is presented. The core part of the algorithm is based on the classical Marquardt-Levenberg (ML) algorithm (Marquardt, 1963, 1970) but associated with a random search loop to wrap the core gradient algorithm for few-layers inversion (previously presented by Schamper and Rejiba, 2011). A vertically constrained (smooth inversion) and a laterally constrained (LCI) versions of the algorithm are used to facilitate the first interpretation of the soundings and to get more consistent resistivity section from a set of neighboring soundings, respectively. The sensitivity of the EMI device to the resistivity as a function of the depth is first analyzed from the inversion of synthetic sounding data to give investigation boundaries. Similar synthetic layered models are then used as benchmarks to test the efficiency of the added random search loop for the estimation of the thickness of a near-surface conductive layer. The sensitivity related to the addition of the radial magnetic component in the inverse problem is then performed on the synthetic models for the same parameters. Forward modeling is also performed for different loop heights to check the sensibility of both H_r and H_z components to this parameter. Finally, a last inversion test of synthetic 2D/3D data is carried out to evaluate the robustness of the present inversion code when the overburden has an oscillating geometry. For in situ illustration, the aforementioned processing tool is applied to data acquired in a watershed basin, located a few kilometers northeast of Rouen (Fontaine-Sous-Préaux, northwest of France). The purpose of the survey was to analyze the application of the PROMIS device in evaluating the thickness of the conductive and clayey overburden located above the karst aquifer embedded inside the Campanien-Santonien chalk formation. The data set presented in this paper was acquired in an area where the thickness varies from a few meters to 10 m. Comparison with the section from an electrical resistivity tomography (ERT) accompanies the analysis of the EMI results.

THEORY

Forward modeling

The response due to the horizontal transmitter loop of the PRO-MIS device can be approximated by the analytical solution for a vertical magnetic dipole (VMD) because the coil spacing is more than 10 times larger than the radius of the emitting loop. Analytical solution for a VMD (and also for electric dipole sources) above a layered ground is detailed in Ward and Hohmann (1988), and also in Wannamaker et al. (1984) and Xiong (1989) for sources and receivers in any layer of the ground. The radial H_r and vertical H_z magnetic components for a VMD above a layered ground can be expressed in terms of Hankel transforms of orders one and zero, respectively:

$$H_{r} = \frac{m}{4\pi} \int_{0}^{\infty} [e^{-u_{0}(z_{s}-z_{r})} - R_{TE}e^{u_{0}(z_{s}+z_{r})}]\lambda^{2}J_{1}(\lambda r)d\lambda$$
$$H_{z} = \frac{m}{4\pi} \int_{0}^{\infty} [e^{-u_{0}(z_{s}-z_{r})} + R_{TE}e^{u_{0}(z_{s}+z_{r})}]\frac{\lambda^{3}}{u_{0}}J_{0}(\lambda r)d\lambda, \quad (1)$$

where *m* is the moment of the transmitter loop $(A.m^2)$,

The variable u_0 in the exponential is defined as $u_0 = \sqrt{\lambda^2 - k_0^2}$ with $k_0^2 = \mu_0 \varepsilon_0 \omega^2 - i \sigma_0 \mu_0 \omega$ as the wavenumber for the air with a magnetic permeability μ_0 , a dielectric permittivity ε_0 , a conductivity $\sigma_0 \approx 0$, and for source emitting at an angular frequency ω .

The terms z_s and z_r are the heights of the transmitter and receiver loops (with *z*-axis oriented downward), respectively.

The terms J_0 and J_1 are the Bessel functions of first kind and orders zero and one, respectively,

The term r is the radial source-receiver distance in the xy plane (m),

and R_{TE} is the reflection coefficient defined as:

$$R_{TE} = \frac{\lambda - \hat{u}_1}{\lambda + \hat{u}_1},\tag{2}$$

with:

$$\hat{u}_{1} = \frac{\hat{u}_{2} + u_{1} \tan h(u_{1}h_{1})}{u_{1} + \hat{u}_{2} \tan h(u_{1}h_{1})} \quad \hat{u}_{n} = \frac{\hat{u}_{n+1} + u_{n} \tan h(u_{n}h_{n})}{u_{n} + \hat{u}_{n+1} \tan h(u_{n}h_{n})}$$
$$\hat{u}_{N} = u_{N}, \tag{3}$$

where $u_n = \sqrt{\lambda^2 - k_n^2}$ is for layer *n* of the *N*-layer 1D model and h_n corresponds to the thickness of layer *n*.

The numerical Hankel transforms are computed using the numerical filters developed by Guptasarma and Singh (1997). These numerical filters have proved to be sufficiently efficient and accurate for frequencies below 100 kHz. Their density is sufficient to handle the singularities that occur due to the expression of $u_i = \sqrt{\lambda^2 - k_i^2}$. For higher frequencies where the propagation part has a more important impact, these numerical filters become less efficient and also inaccurate if their length and/or density are not sufficiently increased. If the logarithmic repartition of the weights of the filters is sufficient for frequencies below 100 kHz, a regular spacing is more necessary when propagation phenomena are present. For frequencies above 100 kHz, other algorithms based on singularities search are preferred to get efficient and accurate Hankel transforms (Aksun and Dural, 2005; Lambot et al., 2007). To improve the convergence of this numerical integration, the primary part of the magnetic field (the term without R_{TE} in factor in equation 1) is removed from the integral and replaced by straightforward equivalent formulas without the Hankel transforms:

$$H_r^P = \frac{m}{4\pi R^3} e^{-ik_0 r} \left[\frac{\Delta x \Delta z}{R^2} \left(-k_0^2 R^2 + 3ik_0 R + 3 \right) \right]$$

$$H_z^P = \frac{m}{4\pi R^3} e^{-ik_0 r} \left[\frac{\Delta z^2}{R^2} \left(-k_0^2 R^2 + 3ik_0 R + 3 \right) + k_0^2 R^2 - ik_0 R - 1 \right],$$
(4)

where R is the source-receiver distance in the 3D space (m).

The difference of altitude between the receiver z_r and the source z_s is defined as $\Delta z = z_r - z_s$. The difference in radial position is expressed as $\Delta x = x_r - x_s$ with x_r and x_s being the positions of receiver and source on the *x*-axis.

These analytical expressions for the electromagnetic field were previously implemented and tested as the background response in the 3D forward modeling code developed by Schamper (2009); Schamper et al. (2011) and based on the method of moments. This 1D forward core is used in the current paper for the inversion. Expressions of H_r and H_z in equation 1 are in the frequency domain and complex. Then, EMI data are further split into two parts: the in-phase (Ip) part, which corresponds to the real part, and the out-of-phase (Op) part, which corresponds to the imaginary part. Both components are normalized by the primary field and expressed as percentages.

Inversion scheme

Core algorithm

It is more difficult to get an objective estimation of the couple resistivity/thickness in the case of a layered earth with a 1D smooth inversion algorithm such as Occam's inversion Constable et al. (1987) than with a few-layer inversion process. In smooth inversion, only resistivities of a large number of thin layers are inverted with vertical smoothing constraints, while a few-layer inversion scheme concerns both resistivity and thickness parameters. For this reason, the few-layer inversion is often preferred for the estimation of parameters, such as thicknesses. In our case, for n_l layers, $2n_l - 1$ parameters are estimated in the logarithmic space, i.e., the thicknesses of the first $n_l - 1$ layers, and the conductivity of each layer.

The classical ML algorithm (Marquardt, 1963, 1970) was first used as the main core of our modified approach: at each iteration *n* of the ML algorithm, the model parameter vector \underline{m}^n was updated to improve the fit between the computed and the field data. This iterative updating process is summarized as follows:

$$[\underline{J}^{n^{T}}\underline{J}^{n} + \lambda^{n}\underline{I}]\Delta\underline{m}^{n} = -\underline{J}^{n^{T}}\underline{r}^{n-1}, \qquad (5)$$

where J^n is the Jacobian, or sensitivity matrix at the n^{th} iteration.

The term λ^n is the damping factor at the n^{th} iteration, \underline{I} is the identity matrix, $\Delta \underline{m}^n$ is the updating vector of the model parameters at the n^{th} iteration, and \underline{r}^{n-1} is the residual vector, i.e., the difference between the forward response (synthetic data) of the estimated model at the *n*th iteration and the field data.

For data quality check and to avoid the difficult interpretation of noisy data during the inversion (which is generally associated with unrealistic results), it is necessary to incorporate the data standard deviation into the inversion process through a data covariance matrix. The data standard deviation is measured during the stacking operation of the measurement device.

To limit the equivalence issues, it is also strongly recommended to use a priori information, which are commonly expressed as a standard deviation with respect to likely values (i.e., in the form of an a priori covariance matrix). To improve the stability and coherence of the inversion results, additional lateral constraints are included (Auken and Christiansen, 2004; Siemon et al., 2009), which corresponds, in practice, to incorporate additional lines in the linear system (see Appendix A for further details). Because the system is solved using the least squares method, these constraints are soft and allow the resistivity values to vary according to the data. However, particular care is required when setting the strength of the constraints to avoid a too smoothed model. Equation 5 is then solved by the damped least-squares method (Menke, 1989):

$$\Delta \underline{\underline{m}}^{n} = -[\underline{\underline{G}}^{n^{T}} \underline{\underline{C}}^{-1} \underline{\underline{G}}^{n} + \lambda^{n} \underline{\underline{I}}]^{-1} \underline{\underline{G}}^{n^{T}} \underline{\underline{C}}^{-1} \underline{\underline{r}}^{n-1}, \qquad (6)$$

where $\underline{\underline{G}}^n$ includes the Jacobian matrix $\underline{\underline{J}}^n$, the additional lines for the a priori information and lateral constraints. The term $\underline{\underline{C}}$ is the covariance matrix, comprising the measurements, the a priori data, and the lateral constraints covariance matrices.

The damping factor λ is adjusted so as to limit the instability associated with solving the linear systems, whose matrix is known to be ill-conditioned (equation 6), (Keys, 1986; Kollias and Anastassiou, 1988; Meju, 1992; Madsen et al., 2004). In the present paper, the approach developed by Madsen et al. (2004) was retained (see lines 20–27 in Appendix A).

The Jacobian matrix \underline{J}^n is evaluated numerically, using finite differences (from the first to the fourth order, depending on the precision required) with respect to the log of the model parameters because 1D forward analytical modeling is run without an excessive computation cost. Further details concerning equation 6 are provided in Appendix A.

For the misfit function Φ mentioned in Appendix A, we considered the data residual for data defined in a linear space as follows:

Data residual =
$$\sqrt{\frac{\sum_{i=1}^{N} \frac{(d_{\text{fwd}}(i) - d(i))^2}{\text{STD}(i)^2}}{N}}$$
, (7)

where N is the number of data points, d_{fwd} is the forward response of the estimated model at the end of the inversion, d contains the data, and STD is the standard deviation of the data.

The data residual has no unit and is always superior or equal to zero. If the data residual is below one, it means that the data are globally well-explained by the estimated model in the margin of error of the data. Between one and 1.5, the data are still quite well-explained, except some data points for which the corresponding forward response is outside of the error bar. Between 1.5 and two, the data are quite badly explained. Above two, the data are generally considered as nonexplained. For synthetic and field EMI data, we always consider a minimum STD of 1% of the primary field (i.e., the unit of the PROMIS data).

 Table 1. Synthetic resistivity models with a conductive nearsurface layer (resistivity/thickness).

M1	M2	M3
70 Ωm/1 m	70 Ωm/1 m	70 Ωm/1 m
20 Ωm/4 m	20 Ωm/10 m	10 Ωm/10 m
120 Ωm	120 Ωm	120 Ωm

Random search loop

A significant issue encountered with gradient methods is associated with local minima. This is usually overcome by quasi-random algorithms, such as Monte-Carlo simulations whose computational cost increases rapidly with the number of parameters. To take advantage of gradient and random methods, we chose to combine the gradient ML algorithm with a local random search procedure. Details of the current ML/random approach are given in Appendix A. At line three of the algorithm, several models (a total of n_{pop}) are randomly generated in the vicinity of an initial model, which is the starting model defined by the user, or the most likely model found during the previous test (a total of n_{test}). Inversion of synthetic data is shown in the next section to present the application of this algorithm.

Depth and top of investigation

Presenting a resistivity sounding or section without mentioning the depth of investigation (DOI) can lead to misinterpretation of deep structures that may finally have no impact on the observed data. Previous works from Oldenburg and Li (1999) and Christiansen and Auken (2010) show techniques to estimate the DOI by analyzing the smooth models obtained from the inversions with two different starting models, or by making a sensitivity analysis of the estimated model which is overdiscretized with very thin layers. Both methods need a sensitivity threshold whose value is estimated using other geophysical surveys. As a first experience using the PROMIS device, we decided to use the method described by Oldenburg and Li (1999). This method consists of performing the analysis of the relative variation of the estimated parameters directly, instead of the forward response. It is then easier to give a first threshold with this method.

First, two smooth inversions are undertaken with two different starting homogeneous models, one conductive and the other one resistive. Then, the two smooth estimated models are compared from the top and the DOI index is computed as follows:

$$R(z) = \left| \frac{\log(\sigma_1(z)) - \log(\sigma_2(z))}{\log(\sigma_1^{\text{ref}}(z)) - \log(\sigma_2^{\text{ref}}(z))} \right|,\tag{8}$$



Figure 1. Standard deviation of the in-phase and out-of-phase parts of the H_r and H_z components, expressed as a percentage of the amplitude of the primary field.

where R(z) is the DOI index as a function of the depth z and σ_1 and σ_2 are the estimated conductivities from the inversions with the starting models 1 and 2. The terms σ_1^{ref} and σ_2^{ref} are the conductivities of the starting reference models one and two.

Once the DOI index exceeds the value of 0.1, the DOI is considered to be reached (this somewhat restrictive value is quite relevant regarding the inversion of synthetic noisy data). Because the highest frequency (56 kHz) is not necessarily large enough to obtain a good definition of the very near surface (depth less than 3 m), we define — analog to DOI — a top of investigation (TOI), which indicates from which depth conductivity values has a real impact on the data. Instead of comparing the two smooth estimated models from the top, the TOI index is computed from the bottom. The same threshold of 0.1 is used.

THEORETICAL STUDIES

DOI and TOI of the ground-based EMI device

A simple synthetic three-layer model (M1 model), consistent with the geologic context of the survey site, is considered. The resistive first layer (with a resistivity of $\rho_1 = 70 \ \Omega$.m and a thickness of $e_1 = 1 \ m$) corresponds to a topsoil, the conductive second layer ($\rho_2 = 20 \ \Omega m$, $e_2 = 4 \ m$) is associated with a clayey overburden, and the resistive last, semi-infinite layer, with a resistivity set to $\rho_3 = 120 \ \Omega m$, represents the unsaturated upper component of a carbonate layer. Two additional models M2 ($\rho_2 = 20 \ \Omega m$, $e_2 = 10 \ m$) and M3 ($\rho_2 = 10 \ \Omega m$, $e_2 = 10 \ m$) are considered to study the effect of a thicker and/or more conductive layer. All three models are summarized in Table 1.

The smooth inversions are performed with noisy synthetic data. For each frequency of the EMI device, the standard deviation of the noise was estimated from field data (Figure 1), and approximated by a normal distribution. A quite large value (close to 3%), still unexplained, is observed at frequency 7040 Hz in the out-of-phase part of the H_z component. However, almost all elements of the data set have a standard deviation below 1.5%.

The inversion results for the three models are shown in Figure 2. For each model, two smooth inversions with either a conductive (10 Ω m) or a resistive (300 Ω m) homogeneous starting model have been considered. In spite of a DOI being estimated around 31 m, it is obvious that the DOI curve starts to increase significantly from 23 m with a larger depth for the M1 model compared to M2 and M3, most likely because of a thinner and less conductive second layer in the M1 model. An increase of the thickness or/and of the conductivity of the overburden layer (i.e., of its conductance) induces a decrease of the DOI which drops to 26 and 19 m for models M2 and M3, respectively. This indicates that the bottom depth of the near-surface conductive layer can be well resolved even for a very low conductivity of 10 Ω m, a thickness of 10 m and a coil spacing of 20 m. The TOI is close to 1 m for all three models, which means that almost no information about the first meter can be retrieved from the PROMIS data. It has to be noted that the offset of 20 m is the smallest coil spacing usable according to the technical specifications from the manufacturer.

Random search algorithm

Synthetic data are generated on the basis of model M1 (Table 1) for thicknesses e_2 ranging between 1 and 10 m. The newly developed inversion algorithm coupled with the local random search is



Figure 2. DOI and TOI estimations for synthetic models M1 (a), M2 (b) and M3 (c) (cf. Table 1). Two homogeneous starting models are considered for the smooth inversion (30 layers, vertical constraint of two), one conductive at $10 \Omega m$, and the other one resistive at $300 \Omega m$. The numbers in brackets indicate the data residuals of the inversions. Noise has been added to the synthetic data using a normal distribution and the estimated standard deviation of field data (Figure 1).



Figure 3. Relative error on the estimated values of the thickness of the conductive layer e_2 for different values ranging from 1 to 10 m ($\rho_2 = 20 \text{ }\Omega\text{m}$). Inversion of H_r and H_z components. The parameters n_{pop} and n_{test} of the inversion algorithm are varied, with the others remaining fixed.

then applied to the synthetic data in Figure 3 (H_r and H_z are inverted jointly). It represents the estimation error of the thickness of the conductive overburden between the model found at the end of the inversion process and the synthetic model used to generate the synthetic data. Figure 3 illustrates the impact of changing both parameters n_{pop} and n_{test} on the estimation error of e_2 . The results show that there is a significant decrease in the error when $n_{test} > 1$, and that the decrease is more significant when n_{pop} is set to a larger value (note that n_{pop} must be greater than one, when $n_{test} > 1$, otherwise it is equivalent to the increase in the maximum number of iterations n, in the absence of a random search). For thicknesses above 7 m, e_2 is always well resolved for $n_{pop} \ge 3$ and $n_{test} \ge 10$. The oscillations observed for the smallest values of n_{pop} and n_{test} indicate that these parameters are not large enough to end the inversion process with the correct thickness systematically.

The change of the resistivity of the conductive layer ρ_2 in the range 10 – 30 Ω m (for a clayey layer) hardly impacts on the



Figure 4. For frequencies ranging from 110 to 56 kHz, and an offset of 20 m, integrated sensitivities of the H_r and H_z components: (a) to the resistivity ρ_2 of the conductive second layer (see $S_{H_{r,z}|\log(\sigma_2)}$ in equation 9), (b) to the thickness e_2 of the conductive second layer (see $S_{H_{r,z}|\log(\sigma_2)}$ in equation 10). The in-phase (*Ip*) and out-of-phase (*Op*) components are shown separately.



Figure 5. Data residual on the H_r (a) and H_z (b) components. All parameters of the layered ground, with the exception of the thickness and the resistivity of the conductive second layer, are fixed. The true model has a layer with a thickness of 4 m and a resistivity of 20 Ω m.

estimated thickness e_2 . In fact, such a conductive layer has a strong impact on the data. A layer with a resistivity of 10 Ω .m is a little bit better resolved due to its higher conductivity contrast with the background. Moreover, Figure 2c has shown that the DOI is below the bottom depth of a 10 m thick layer with a resistivity of 10 Ω .m.

Multicomponent inversion $(H_r \text{ and } H_z)$

Sensitivity analysis $H_z - H_r$ versus $H_z - H_z$

The synthetic model M1 (Table 1) is considered as the basis model for the analysis. The sensitivities of H_r and H_z to the resistivity, and to the thickness of the conductive layer are defined as follows:

$$S_{H_{r,z}|\log(\sigma_2)} = \sqrt{\sum_{i=1}^{10} \left| \frac{\partial H_{r,z}(\rho_1, \rho_2, \rho_3, e_1, e_2, f_i)}{\partial \log(\sigma_2)} \right|^2} \quad (9)$$

$$S_{H_{r,z}|\log(e_2)} = \sqrt{\sum_{i=1}^{10} \left| \frac{\partial H_{r,z}(\rho_1, \rho_2, \rho_3, e_1, e_2, f_i)}{\partial \log(e_2)} \right|^2}, \quad (10)$$

where f_i is one of the 10 frequencies recorded by the EMI device (Hz).

Because the log of the parameters are estimated during the inversion, and because the conductivity rather than the resistivity is inverted for EMI data, the derivatives are estimated with respect to $log(\sigma_2)$ for the resistivity ρ_2 and with respect to $log(e_2)$ for the thickness e_2 . The sensitivities of the H_r and H_z receivers to ρ_2 and e_2 are shown in Figure 4.

Regarding the sensitivity to ρ_2 (Figure 4a), both components have very similar sensitivities for very low resistivities below 3 Ω m. Above this value, the sensitivity of $Ip(H_r)$ is always above the ones of the two parts of H_z . The sensitivity of $Op(H_r)$ becomes higher from a resistivity of about 12 Ω m.

For the sensitivity to e_2 (Figure 4b), the $Ip(H_r)$ and $Op(H_r)$ sensitivities are higher than the ones of the two parts of H_z until thicknesses of 15 and 8 m, respectively. Above 15 m, both $Ip(H_z)$ and $Op(H_z)$ have a better sensitivity compared to H_r , meaning that the H_z component is particularly sensitive to the deeper layers compared to H_r , which shows a better sensitivity to a near-surface conductive overburden.

The data residual defined in equation 7 is plotted (Figure 5) as a function of thickness and resistivity of the conductive second layer with model M1 (Table 1) as true model. As mentioned before, a minimal STD of 1% of the primary field is considered for all frequencies. An elongated zone is observed, in which the data residual is very small, for H_r and H_z . The narrow area of low-residual is characterized by sharper variations in the vicinity of the true model for H_r , whereas the corresponding ellipsoid is less extended along its principal axis for H_z . These complementary characteristics associated to both components should improve the constraints required during the inversion process. The product of the resistivity by the thickness is better bounded with H_z , whereas a better convergence to the true model is obtained with H_r .

Inversion of H_r and H_z

Similarly to the example shown in Figure 3, the inversion algorithm is applied to synthetic data in Figure 6. In this simulation, the algorithm's parameters remain constant for all inversions ($n_{pop} = 3$, $n_{test} = 10$), and correspond to three different cases: H_r only, H_z only, and both H_r and H_z . Because no random noise is added to the synthetic signals, the estimation error is very low. In addition, the inversion of H_r and H_z components allows the estimation error to be decreased by a factor of one decade, when compared to the inversion of the H_z component only.



Figure 6. Relative error on the estimated values of the thickness of the conductive second layer e_2 for different values ranging from 1 to 10 m ($\rho_2 = 20 \Omega$ m). Different data sets are considered, with only one component H_r or H_z , or with H_r and H_z components. Synthetic data are considered here without noise. Light gray curves are tendencies to help the reading. The parameters for the random search are set to $n_{pop} = 3$ and $n_{test} = 10$.



Figure 7. Relative error on the estimated values of the thickness of the conductive second layer e_2 for different values ranging from 1 to 10 m ($\rho_2 = 20 \,\Omega$ m). Different data sets are considered, with only one component H_r or H_z or with H_r and H_z components. Synthetic data are considered here with random noise and a stack of 10. Light gray curves are tendencies to help the reading. The parameters for the random search are set to $n_{pop} = 3$ and $n_{test} = 10$.

As for the DOI/TOI estimation in Figure 2, noise is added to the synthetic data. The same inversion process as for Figure 6 with noncontaminated data is then applied. The errors corresponding to the estimation of the thickness of the conductive layer are plotted in Figure 7. Except for $e_2 > 7$ m, the inversion using H_r and H_z components improves significantly the confidence of the thickness estimation, i.e., with errors one decade below those associated with thickness determinations based on the use of a single component. As mentioned for the test of the random search loop, the change of the resistivity ρ_2 in the range $10 - 30 \ \Omega m$ does not imply noted differences in the estimation of the thickness e_2 .

Consideration of the heights of the loops

Figure 8 shows the impact of the heights of the loops on both components H_r and H_z above the synthetic model M1 (Table 1). Three different height configurations are studied and noted as Conf. 1, Conf. 2, and Conf. 3 (Figure 8). The configuration Conf. 1 concerns the height change of transmitter and receiver coils, located 50 cm above a reference configuration where coils are held 1 m above the ground. This synchronized change of height has no effect on the H_r component and has a little effect on the two highest frequencies of the out-of-phase part of the H_z component.



Figure 8. Effect of the heights of the loops on the forward response for H_r and H_z components (left and right columns, respectively), h_{Tx} corresponds to the height of the transmitter loop, and h_{Rx} to the height of the receiver loop. The offset considered between the two loops is 20 m. The layered earth is model M1 (Table 1).

If the height of only one of the two coils is changed (Conf. 2 and 3 of Figure 8), the effect is significant on the in-phase part of H_r whose curve is shifted at all frequencies. For these last two configurations, the change in the out-of-phase part of H_z is smaller than for Conf. 1. For Conf. 2 and Conf. 3, the change for the $H_z - H_z$ configuration is identical because of the equivalence between source and receiver.

Consequently, the heights of the loop are further considered as parameters during the inversion. The number of parameters for each few-layer model thus increases from $2n_l - 1$ to $2n_l + 1$ for a few-layer inversion.

Single-site and laterally constrained 1D inversion above a nonlayered ground

The PROMIS device can be easily carried by two persons to make several neighboring soundings, and to get vertical sections of resistivity. In the two previous sections, the lateral smoothing constraints were not used, and we propose in this section to analyze the inversion of a profile above a structure close but not identical to a one-dimensional model. A cross section of this model is shown in Figures 9 and 10a. The model contains a conductive layer of $20 \,\Omega m$ embedded inside a homogeneous ground of $50 \,\Omega m$, and whose



Figure 9. Forward response of the synthetic 2D/3D model. Dashed lines correspond to the response of the homogeneous background only, and solid lines to the response of the ground including the conductive layer. Only the five highest frequencies of the PROMIS device are displayed. The offset considered is 20 m. The 28 kHz frequency of $Op(H_z)$ is displayed in bold to distinguish it from the other frequencies.

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Figure 10. Inversion of synthetic data above a nonlayered ground: (a) cross-section of the true model (b) starting model, (c) 1D smooth inversion with 30 layers (vertical constraints of 2), (d) 1D inversion without random search, (e) 1D inversion with a random search ($n_{pop} = 4$, $n_{test} = 10$), (f) LCI of all soundings with lateral constraints of 1.4, (e) the "best" model obtained if all resistivities are fixed to the values of the true model.

depth varies along the profile. The conductive 3D layer is sufficiently extended perpendicularly to the profile to be considered as a 2D model, and all potentially out-of-profile lateral effects can be neglected. The synthetic data (Figure 9) were generated using the 3D code EM_MOM based on the Method of Moments, which has been first used to model the response of deep targets (Schamper, 2009; Schamper et al., 2011). This method allows the discretization of the 3D target embedded in a layered background whose response is computed analytically. The cells of the current modeling have a lateral size of 5 m and a thickness of 1 m. A single offset of 20 m is considered, and the two loops are moved above the structure at a height of 1 m. The middistance between the coils is considered as the position of each sounding. As shown in Figure 9 only frequencies above 3.5 kHz start to show differences due to the presence of the conductive layer with an effect increasing with the augmentation of the frequency. On both sides and at the middle of the 3D structure, the total forward response (background and scattering response from the 3D target) is constant along the profile, which means that the response of the 3D model is equivalent to the one of a 1D structure. At other positions on the profile, 2D/3D effects are clearly visible where the depth of the conductive layer is changing.

The starting model is the same for all soundings along the profile (Figure 10b), and consists in a homogeneous earth of $60 \Omega m$. Both H_r and H_z components are inverted jointly. All inversion parameters are the same for the four different cases: the 1D smooth inversion of each sounding (Figure 10c; with vertical constraints of 2.0); the 1D inversion of each sounding without random search (Figure 10d); the 1D inversion of each sounding with random search (Figure 10e, with $n_{pop} = 4$ and $n_{test} = 10$); and the LCI of all soundings with soft lateral constraints activated (Figure 10f; with lateral constraints of 1.4). The softening lateral constraints of the LCI tend to limit the variation of the resistivity values and of the interface depths from one sounding to another. The value of 1.4 (equivalent to a variation of 40%) is not very strong and is sufficiently loose to avoid a too laterally smoothed section.

In the present case, where the thickness of the conductive layer is constant and fixed to 4 m, there is almost no difference between the 1D inversions with a few-layer model (Figure 10d and 10e). For 1D few-layer inversions, the oscillations of interfaces are more pronounced at positions -75 m and +75 m compared to the LCI (Figure 10f) for which lateral constraints tend to impose smooth lateral variations.

The thickness of the conductive layer is well estimated for these

three inversions (Figures 10d, 10e, and 10f), whereas the depth is underestimated by 1 m on both sides of the profile. As demonstrated by the TOI estimation in Figure 2, the first meter cannot be resolved with the present configuration. In the present case, the inverted resistivity value remains close to the value of the initial guess in Figure 10b.

For the resistivity section displayed in Figure 10g, the resistivities are fixed to the values of the true model, while the thicknesses are not constrained during the inversion process. The depth of the conductive layer is then well determined for this case. There is still an overestimation of the depth of the conductive layer by 0.5 m

at the center of the profile where the interface is closest to the surface (Figure 10g). A slight difference in the forward response between the 3D modeling and the equivalent 1D forward modeling suggests that this small offset could be explained by the vertical size of the cells of the 3D modeling, which has been set to 1 m. A finer vertical discretization should have been necessary to get a better match near the ground surface.

For the 1D smooth inversion (Figure 10c), the depths of the layers are fixed, and only resistivities are inverted. Vertical constraints on the resistivities are set to smoothen the vertical variations. For this type of inversion, there are n_1 estimated parameters, the number of layers being generally larger than 10. In the present case, the number of layers is 30, and their thicknesses increase logarithmically as a function of the depth. The application of the 1D smooth inversion on this synthetic example gives an informative picture (Figure 10c) compared to the true model section (Figure 10a). Although this type of inversion cannot give precise locations of the interfaces due to the vertical constraints on the resistivities, it remains a first step of the interpretation by giving a preliminary result without a priori knowledge.

APPLICATION TO THE KARST SITE OF FONTAINE-SOUS-PRÉAUX (FRANCE)

The site

In Upper Normandy (northwest France), the Fontaine-Sous-Preaux basin provides 60% of Rouen's population (around 500,000 including the suburbs) with water from karst aquifers. The site has numerous sinkholes and is composed of a topsoil layer, a clayey overburden, and then a chalk plateau, which makes this basin vulnerable to pollution from infiltration. Depending on the groundwater level, the upper unsaturated part of the karst reservoir could be expected to have quite a high electrical resistivity, particularly when compared with the resistivity of the clayey cover, which is composed of silt and flint clays (from the top of the chalk up to the ground surface). Previous geotechnic soundings and geologic observations (Leclerc, 2008) indicate a thickness in the range between a few meters and 35 m for both of these conductive formations. The clayey overburden, composed of a loess layer with a resistivity about $10 - 20 \Omega m$ and of a flint clayey layer with a resistivity about $30 - 40 \Omega m$ (alteration of the top of the chalk layer), must be suitably characterized to study the karst aquifer's vulnerability to infiltration.



Figure 11. 2D model obtained from the inversion of an ERT (Wenner-Schlumberger), using the Res2dinv software (robust inversion with the L1 norm). The location of the four PROMIS soundings are shown on this ERT.



Figure 12. PROMIS data with comparison with forward responses from the smooth inversion of the PROMIS soundings and the smooth models taken from the ERT. From the top to the bottom: soundings S1 to S4 (see positions in Figure 11). Left column: data and forward responses of the H_z component. Right column: data and forward responses of the H_z component. The three highest frequencies in the in-phase part of the H_z component are not considered during the inversion of the PROMIS soundings.

As part of this study, the site was also investigated using ERT and the Wenner-Schlumberger configuration. The ERT measurements were carried out with the SYSCAL Pro (from Iris Instruments). The final ERT of 191 m is composed of three successive roll-allong ERTs, each one composed of 96 electrodes spaced by 1 m. The data have been inverted using the software Res2dinv (Loke and Barker, 1996), and are displayed in Figure 11. From this section we can distinguish four different layers: the first one with a resistivity about 70 Ω m and a small thickness of 0.5 m (which is likely related to the top soil), a very conductive second layer with resistivity around $10 - 20 \Omega m$ and a mean thickness of an order of 2 m (loess layer), then a conductive layer of $30 - 40 \ \Omega m$ and a mean thickness of 4 m (flint clay), and a last layer with a resistivity above 70 Ω m (chalk). This ERT section has been considered as sufficiently reliable to be defined as our reference for the comparison with the PROMIS soundings (S1 to S4) whose locations are pointed at the top of the section (Figure 11).

The data

Data from the four PROMIS soundings (S1 to S4 in Figure 11) are displayed in Figure 12 with the initial standard deviation coming from the measurements. The forward EMI responses from the 1D resistivity models taken from the ERT are added to the data curves. These ERT curves generally follow EMI data closely, except for sounding S2, especially in the three highest frequencies of the in-phase part of the H_z component ($Ip(H_z)$), which are oscillating

abnormally. Differences at the highest frequencies of the out-of-phase part of the H_r are also observed for S2, but are much less pronounced. A similar mismatch is observed in the immediate neighboring sounding S3, but only in the two highest frequencies of $Ip(H_z)$. For soundings S1 and S4, the mismatch in $Ip(H_z)$ concerns the highest frequency only. According to these observations, which indicate a possible capacitive coupling (regarding the oscillations at sounding S2), it has been decided to ignore the three highest frequencies of $Ip(H_z)$ for the inversion of the EMI data.

The purpose of the ERT was to check the initial factory calibration of the PROMIS device and to verify if supplementary calibration was needed as proposed by Lavoué et al. (2010). They define linear dependencies (plus a constant shift) between the apparent resistivity coming directly from the EMI device and the one from the ERT for each frequency separately. Unfortunately, the limited number of EMI soundings along the ERT profile did not allow us to estimate such dependencies accurately. However, the present factory calibration was sufficient for the topic of this paper.

Smooth inversion

To analyze the EMI data, we first apply a smooth inversion to the four PROMIS soundings (S1 to S4) to compare the smooth models with the ERT section (Figure 11). The results are

displayed in Figure 13. Components H_r and H_z are considered. The number of layers is set to 30, with thicknesses increasing logarithmically down to 30 m, the depth of the last interface. The local smooth curve from the ERT section is drawn with a thick black line and will be used as a reference.

To check the reliability of the forward and inverse modelings, the EMI data are also inverted with *em1dinv* code developed by Christiansen and Auken (2008). This code also allows the joint inversion of the two components. A particular precaution has been considered regarding the inversion parameters so that they are pretty much the same for both codes. A first inversion was made considering the initial data STD from the measurements, plus a margin error of 1% of the primary field to avoid artifacts due to small oscillations in the lowest frequencies where the secondary magnetic field is close to 0%. Both codes give almost the same resistivity curves, which end with larger resistive values compared to the ERT (Figure 13). This could be explained by the lower sensitivity of the EMI device to resistive layers, which makes this part of the resistivity curve sensitive to small noise level.

A supplementary STD factor of 0.05 (a relative error of 5%) was considered in a second inversion run, which allows the reduction of the deepest resistivity values for three of the four soundings. The data residual also drops below one for all soundings, which indicates that the data are well explained in this estimated margin of error. Regarding the difference of height between the transmitter and the receiver loops, this parameter is well determined because it induces a global shift to all frequencies of $Ip(H_r)$ (Figure 8),



Figure 13. Smooth inversion of the PROMIS soundings, S1 to S4 (see positions in Figure 11). Inversion configuration: 30 layers down to 30 m, vertical constraints = 2. The three highest frequencies in the in-phase part of the H_z component are not considered during the inversion of the PROMIS soundings. A supplementary STD factor of 0.05 has been considered to get a lower data residual and to better explain the error margin of the data.

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including the lowest ones, which are always close to 0%, regardless of the ground resistivity. An average difference of 30 cm has been found for the four PROMIS soundings, which is in accordance with the field work.

Figure 13 shows the same inflection point between the conductive overburden and the resistive chalk for ERT and EMI resistivity curves. Despite the lower resistivity values obtained for the deeper layers of the EMI models by considering a higher STD on the data, EMI resistivities are generally higher compared to the ERT, except in the first meters of the clayey overburden where EMI resistivity are often lower than ERT resistivity. These differences can be explained by several factors: (1) the calibration may be not sufficiently accurate to get the same resistivity levels; (2) the two methods do not integrate the underground resistivity in the same manner, EMI method being more sensitive to conductive layers compared to DC method; (3) with different sensitivity imprints, the two methods may be also differently affected by lateral variations; (4) the differences could be explained by a biaxial anisotropy of the layers. Sinha (1968) demonstrated that the biaxial anisotropy cannot be detected with a magnetic dipole source in the air, and that systems with such a source are only sensitive to the horizontal resistivity. However, the ERT with buried electric dipoles is sensitive to the biaxial anisotropy.

Regarding the number of layers, the EMI results generally show two layers only (Figure 13), a first one conductive, which includes the loess and the flint clayey layers, and a second one resistive corresponding to the chalk. The resistive first layer of less than 1 m



Figure 14. Few-layer single-site inversions ($n_{pop} = 3$, $n_{test} = 10$) and 1D LCIs of the PROMIS soundings, S1 to S4 (see positions in Figure 11). Inversion configuration: three-layer or four-layer model, both H_r and H_z components, the three highest frequencies in the in-phase part of the H_z component are not considered during the inversion of the PROMIS soundings.

thickness in the ERT section is almost invisible, which is coherent with the value of the TOI. The estimation of the TOI and DOI is made following the same methodology as the one used for the synthetic data in Figure 2. The only difference is the threshold, which has been set to 0.2 instead of 0.1 for the noisy synthetic data.

The average thickness of the clayey overburden determined from the ERT section is similar to the thickness that has been interpreted from the EMI data (Figure 13). In the ERT section of Figure 11, the thickness of the clayey overburden is clearly smaller (≈ 4.5 m) where sounding S1 is located compared to the rest of the profile (≈ 7 m). This difference is clearly observed in the interpreted resistivity curve of sounding S1 compared to the other soundings (Figure 13).

Few-layer inversion

Regarding the previous interpretation from smooth inversion, a three-layer model is used with a thin, highly-resistive cover layer that is poorly defined by the EMI data. The results of the inversion are displayed in Figure 14. For the single-site (1D) inversion, the parameters of the random search loop have been set to $n_{pop} = 3$ and $n_{test} = 10$, values that have shown good performance in the benchmark test of Figure 3. A supplementary STD factor of 0.05 is also considered. As four layers are distinguished in ERT section (Figure 11), a similar 1D inversion was run with four layers. The final data residual is the same for three-layer and four-layer inversions, and is also identical to the data residual of the previous smooth in-

version (the one with the supplementary STD factor of 0.05 in Figure 13). This indicates that no more than three layers are necessary to explain EMI data. One-dimensional LCIs have been also undertaken with three or four layers (Figure 14) with lateral constraints set to 1.4. The model curves are almost identical from the surface to the bottom of the conductive layer compared to the 1D single-site inversion with the same number of layers. The resistive value of the last layer is, however, closer to the ERT value with the 1D LCI. It is likely due to the stronger weight obtained for the resistive last layer thanks to the lateral constraints between the soundings.

CONCLUSION

Results from our theoretical sensitivity analysis, and from the inversion of synthetic data, illustrate the advantages of a multicomponent inversion of EMI data. They also show that a significant improvement is achieved in terms of convergence, leading to a more confident interpretation. By reducing the estimation error by a factor of at least one decade, data inversion of H_r and H_z definitely improves the quality of the inversion, when compared with that achieved with the inversion of any single component. This result is particularly pronounced in the case of a conductive horizon lying above a more resistive layer, in which case H_r appears to be more sensitive to the thickness of a near-surface conductive layer than H_{z} .

We have developed an interpretation tool that allows to invert the multifrequency and multicomponent data of a EMI device (here, the PROMIS device developed by Iris Instruments). Different types of inversion can be used: few-layer and smooth 1D inversion for the interpretation of local soundings, and LCI for the modeling of several neighboring soundings. For the 1D single-site inversion, we add an algorithm of random search which, combined with the ML algorithm, helps to find the best minimum when a priori information about resistivities and thicknesses is missing, or when repetitive information along a profile cannot be used. The 1D LCI of a 3D synthetic model has shown the value of this type of inversion for getting a more geologically reliable model at a reasonable computation cost.

Field measurements with the EMI device PROMIS have been undertaken above a karstic area in France where the estimation of the thickness of the clayey overburden is of principal concern for the vulnerability study of the karstic aquifer to the infiltration of pollutants. Despite the limited number of EMI soundings, the results obtained with the interpretation tool have shown fairly good agreements with an ERT perfomed at the same location. The initial factory calibration was sufficiently accurate for this study, but better calibration is required in a production survey. In addition, highly probable EM coupling have been observed at the highest frequencies of the in-phase part of the H_z component with an offset of 20 m. Nevertheless, the inversion without considering the three highest frequencies of the in-phase part of H_z gives satisfying results. Although the two clayey layers constituting the overburden cannot be distinguished from one to the other, the thickness of the clayey overburden can be estimated in the field with the present configuration of the PROMIS device and the presented interpretation tool.

ACKNOWLEDGMENTS

We thank anonymous reviewers for their very constructive remarks, which have improved the quality of this paper. We wish to thank Iris Instruments, and Orlando Leite in particular, for lending us the PROMIS instrument. We also wish to thank Matthieu Fournier and Amer Mouhri, from the University of Rouen, and Véronique Lecomte from la CREA of Rouen (Urban Commmunity of Rouen-Elbeuf-Austreberthe), for their assistance and suggestions. Alain Tabbagh from Sisyphe laboratory is thanked for his pertinent and constructive remarks. Esben Auken from HGG group in Aarhus University is thanked for allowing the use of the *em1dinv* code for the cross checking. Finally, we wish to thank Marie Bergeron, a M1 trainee, for her assistance during the field measurements.

APPENDIX A

MARQUARDT-LEVENBERG METHOD COUPLED WITH A RANDOM SEARCH

- 1: For test = 1 to n_{test} do
- 2: $m_0 \leftarrow \underline{m}_{\text{test-1}}$
- 3: Generation of n_{pop} random models in the vicinity of $\underline{m_0}$ Decreasing the risk of a local minimum solution
- 4: For mod = 1 to n_{pop} do

5: n = 1

(continued)

6:	$r_0 \leftarrow A(m_0^{\text{mod}}) - \underline{d}$
7:	$\overline{\lambda^1}$ defined by the user, generally ≥ 1
8:	ν initially defined by the user, >1
9:	While $n \le n_{\max}$ & $\ \underline{r}^{n-1}\ ^2 \ge \epsilon$ do
10:	Compute: $\underline{J}^n \leftarrow \partial_{m_i} a_k$
11:	decrease _{Φ} = 0; borns_exceeded = 1
12:	While decrease $\Phi = 0$ or borns_exceeded = 1 do
13:	decrease _{Φ} = 0; borns_exceeded = 1
14:	Solve linear system using the least-squares method: $\Delta \underline{m}^{n} = -[\underline{\underline{G}}^{n^{T}} \underline{\underline{C}}^{-1} \underline{\underline{G}}^{n} + \lambda^{n} \underline{\underline{I}}]^{-1} \underline{\underline{G}}^{n^{T}} \underline{\underline{C}}^{-1} \underline{\underline{r}}^{n-1}$
15:	$\underline{m}^{n} \leftarrow \underline{m}^{n-1} + \Delta \underline{m}^{n}$
16:	if \underline{m}^n inside limits then
17:	borns_exceeded = 0
18:	$\underline{r}^{n} \leftarrow A(\underline{m}^{n}) - \underline{d}$
19:	$\phi^n \leftarrow f_{\phi}(\underline{r}^n)$ Evaluation of the misfit function
20:	$g \leftarrow \frac{\Phi^{n-1} - \Phi^n}{1/2m^{n-1} + m^{n-1}}$
21:	If $g < 0$ then
22:	$\lambda^n \leftarrow \lambda^n \times \nu; \nu \leftarrow \nu \times 2$
23:	$\underline{m}^{n} \leftarrow \underline{m}^{n-1}$
24:	else
25:	$decrease_{\Phi} = 1$
26:	$\lambda^{n+1} \times \leftarrow \lambda^n \max(1/3, 1 - (2g - 1))$
27:	We can now proceed to the next iteration
28:	end if
29:	else
30:	$\lambda^n \leftarrow \lambda^n \times \nu; \ \nu \leftarrow \nu \times 2$
31:	$\underline{m}^{n} \leftarrow \underline{m}^{n-1}$
32:	end if
33:	end while
34:	$n \leftarrow n + 1$
35:	end while
36:	end for
37:	Select the most likely model, from the n_{pop} model parameter vectors $\leftarrow \underline{m}_{test}$
38:	end for

where n_{test} is the number of times several starting models are randomly constructed around a starting model m_0 .

The vector $m_{\text{test-1}}$ is the most likely model from the previous test (when test = 1, it corresponds to the initial model defined by the user).

The term n_{pop} is the number of randomly generated models.

The term n is the number of iterations for each inversion from a starting model.

The term r_0 is the initial residual vector.

The term $\overline{A}()$ is the forward modeling operator.

The term $\underline{m_0}^{\text{mod}}$ is the mod^{*th*} initial model among the generated population.

The term \underline{d} is the data vector,

The term λ is the damping factor whose first value λ^1 is defined by the user.

The term ν is a parameter used to tune the damping factor (λ) variations,

 $n_{\rm max}$ is the maximum number of iterations for each inversion.

The term e is the stopping criterion on the L2 norm of the residual vertor r^{n-1} .

The term \underline{J}^n is the Jacobian, or sensitivity matrix, which is numerically estimated using finite differences, and whose size is $[n_d, n_p]$, with n_d the number of data records and n_p the number of inverted parameters. For a layered earth model with a number of n_l layers and a number of n_s soundings belonging to the same profile, $n_p = n_s \times (2 \times n_l - 1 + 2)$ if we take into account the electrical resistivities, the thicknesses and the heights of the emitting and receiving coils.

The term $\underline{\underline{G}}^n$ is the system matrix composed of the Jacobian matrix $\underline{\underline{J}}^n$, a $[n_p, n_p]$ identity matrix for the a priori constraints $\underline{\underline{I}}$, and the matrix concerning the lateral constraints $\underline{\underline{L}}$:

$$\underline{\underline{G}}^{n} = \begin{vmatrix} \underline{\underline{J}}^{n} \\ \underline{\underline{I}} \\ \underline{\underline{L}} \end{vmatrix} .$$
 (A-1)

The lateral constraints are applied on the resistivities and the thicknesses of the geologic layers. Then, for a layered earth model with a number of n_l layers and a number of n_s soundings corresponding to the same profile, the matrix \underline{L} has a size of $[(n_s - 1) \times (2 \times n_l - 1), n_p]$. In the current version of the inversion code, we only consider constraints between two consecutive soundings, which finally leads to lateral constraints for all soundings of the same profile. The matrix \underline{L} has a structure similar to:

$$\underline{\underline{L}} = \begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 & -1 \end{vmatrix}.$$
 (A-2)

The term $\underline{\underline{C}}$ is the covariance matrix, which is composed of the data covariance $\underline{\underline{C}}_{data}$ ($[n_d, n_d]$), the a priori covariance $\underline{\underline{C}}_{apriori}$ ($[n_p, n_p]$), and the lateral constraints $\underline{\underline{C}}_L$ ($[n_s \times 2 \times n_l, n_s \times 2 \times n_l]$) matrices:

$$\underline{\underline{C}} = \begin{vmatrix} \underline{\underline{C}}_{\text{data}} & 0 & 0 \\ 0 & \underline{\underline{C}}_{\text{apriori}} & 0 \\ 0 & 0 & \underline{\underline{C}}_{L} \end{vmatrix}.$$
(A-3)

Here, all covariance matrices are assumed to be diagonal, which enables straightforward computation of the inverse matrix \underline{C}^{-1} .

The term g is the gain factor used to determine the efficiency of the updating,

The term decrease_{Φ} and borns_exceeded are the loop exit conditions, on a decrease in the misfit function, and on the parameters' bounds, respectively, as defined by the user, and $\Delta \underline{m}^n$ is the updating model parameter vector.

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