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Strictly horizontal lateral parameter correlation for 1D inverse modelling of large datasets

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ABSTRACT

This paper presents new developments to the lateral parameter correlation method, a method that can be used to invoke lateral smoothness in model sections of one-dimensional inversion models. The lateral parameter correlation method has three steps. First, all datasets are inverted individually. Next, a laterally smooth version of each model parameter is found by solving a simple constrained inversion problem by postulating identity between the uncorrelated and correlated parameters and solving the equations including a model covariance matrix that ensures lateral smoothness. As a final step, all sounding data are reinverted with the correlated model parameter values as *a priori* values to produce models that better fit the data. Because the method separates inversion and correlation, it is much faster than methods where the inversion and correlation are solved simultaneously. The new development to the lateral parameter correlation method presented in this paper is an option to perform a strictly horizontal correlation, thereby avoiding the model artefacts sometimes seen when correlating along layers, namely that formations tend to follow the topography. Furthermore, a solution to the intractable computation times arising with large datasets is formulated, employing a tessellation of the plane and an averaging scheme within the subareas that reduces the size of the numerical lateral parameter correlation inversion problem while maintaining correct correlation within a very large area. In a field example, correlation along layers and the new strictly horizontal correlation are compared, and it is demonstrated that the horizontal correlation removes the above mentioned artefacts that can appear when correlating along layers. The lateral parameter correlation method is very flexible and is capable of correlating models from inversion of different data types, including information from boreholes, and it lends itself easily to “embarrassingly parallel” computation.

INTRODUCTION

Over the past half century, electrical and electromagnetic methods have been used to solve a huge variety of environmental and hydrogeophysical problems (Fitterman 1987; Sandberg and Hall 1990; Taylor, Widmer, and Chesley 1992; Albouy *et al.* 2001; Sørensen *et al.* 2005; Auken *et al.* 2006; Macnae 1997; Binley *et al.* 2015). More and more, single-site applications of various methods have given way to the use of continuous, measure-while-moving methods, both ground based and airborne, and it has become economically possible to investigate extensive areas with fairly dense coverage. Continuous geoelectrical methods are presented by Sørensen (1996) and by Panissod, Lajarthe, and Tabbagh (1997). Airborne systems in both frequency and time domains, today often helicopter-borne, are finding increased use. The frequency-domain helicopter-borne electromagnetic (EM) method (Sengpiel and Siemon 2000; Siemon 2001; Tølbøll and Christensen 2006) and airborne transient systems (Balch *et al.* 2002; Eaton *et al.* 2002; Macnae 1997), including the recent SkyTEM system (Sørensen and Auken 2004), have been used for mining purposes, in hydrogeophysical and geotechnical investigations, and for mapping salt water intrusion, capping clays, etc.

Modern, continuous electrical and electromagnetic methods are capable of recording very large datasets, and although 3D inversion of large EM datasets is now an option (Yang and Oldenburg 2012; Oldenburg, Haber and Shekhtman 2013; Yang, Oldenburg, and Haber 2014), standard inversion methodology still relies mainly on 1D models. In environments where the lateral rate of change of resistivity is small, 1D inversion is justified, which is often the case in layered sedimentary environments. In recent years, Markov chain Monte Carlo methods have been used in interpreting large EM datasets (Malinverno 2002; Sambridge *et al.* 2006; Minsley 2011; Brodie and Sambridge 2012).

Because of the local character of the data noise in space and time, individual inversion of sounding data does not ensure lateral continuity of the model sections. However, when lateral changes in conductivity are expected to be small, it is reasonable

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to impose continuity by lateral correlation of the models. Several techniques for lateral correlation of 1D earth models have been presented in the literature (e.g., Gyulai and Ormos 1999; Auken and Christiansen 2004; Auken *et al.* 2005; Brodie and Sambridge 2006; Viezzoli *et al.* 2008; Viezzoli, Auken, and Munday 2009).

In this paper, I present new developments within the framework of the lateral parameter correlation (LPC) method presented earlier by Christensen and Tølbøll (2009). The basic characteristics of the method are maintained. First, all data sets are inverted individually. Next, a laterally smooth version of each model parameter is found by solving a simple, linear constrained inversion problem. Identity is postulated between the uncorrelated and correlated parameters, and the equations are solved including a model covariance matrix that ensures lateral smoothness. The correlated models do not fit the data as well as the uncorrelated inversion models; therefore, as a last step, all sounding data are inverted again to produce models that better fit the data, now constrained to the correlated model parameter values as *a priori* values. In this paper, the method is extended to perform a strictly horizontal correlation, thereby avoiding the model artefact sometimes seen when correlating along layers, namely that the correlation forces formations to follow the topography.

Furthermore, a solution to the intractable computation times arising with large datasets is formulated through a newly developed tessellation of the plane into averaging cells of increasing size with distance from a central sounding. In this way, the inversion problem of the correlation is divided into many small problems that are easily solved while preserving all relevant information in the correlation. The methodology and performance are demonstrated through a field example of helicopter-borne transient electromagnetic data, where correlation along layers and strictly horizontally are compared, but the approach is general and can be used with any continuous method.

INVERSION AND LATERAL CORRELATION

Inversion methodology

The 1D inversion formulation used with both the airborne electromagnetic (EM) data of the field example and in the inversion involved in the lateral correlation is a well-established iterative damped least squares approach (Menke 1989) with models consisting of horizontal, homogeneous, and isotropic layers. The model update at the *n*th iteration is given by

$$\mathbf{m}_{n+1} = \mathbf{m}_n + \left[\mathbf{G}_n^T \mathbf{C}_{obs}^{-1} \mathbf{G}_n + \mathbf{C}_{prior}^{-1} + \frac{1}{\sigma_m^2}\mathbf{C}_m^{-1}\right]^{-1}, \tag{1}$$

$$\left[\mathbf{G}_n^T \mathbf{C}_{obs}^{-1}\left(\mathbf{d}_{obs} - \mathbf{g}(\mathbf{m}_n)\right) + \mathbf{C}_{prior}^{-1}\left(\mathbf{m}_{prior} - \mathbf{m}_n\right)\right], \tag{2}$$

Where m is the model vector containing the logarithm of the model parameters, G the Jacobian matrix containing the derivatives of the data with respect to the model parameters, T the matrix transpose, C_{obs} the data error covariance matrix, C_{prior} the covariance matrix of the prior model, C_m a model covariance matrix imposing the vertical smoothness constraints of the multilayer models, σ_m the standard deviation assigned to the model covariance matrix determining the strength of the smoothness constraint, d_{obs} the field data vector, $g(m_n)$ the nonlinear forward response vector of the *n*th model, and m_{prior} the prior model vector. In this study, as is most often the case, the data noise is assumed uncorrelated, implying that C_{obs} is a diagonal matrix. The model parameter uncertainty estimate relies on a linear approximation to the posterior covariance matrix $\mathbf{C}_{est}$ given by

$$\mathbf{C}_{est} = \left[\mathbf{G}^T \mathbf{C}_{obs}^{-1} \mathbf{G} + \mathbf{C}_{prior} + \frac{1}{\sigma_m^2}\mathbf{C}_m^{-1}\right]^{-1}, \tag{3}$$

Where $\mathbf{G}$ is based on the model achieved after the last iteration. The analysis is expressed through the standard deviations of the model parameters obtained as the square root of the diagonal elements of $\mathbf{C}_{est}$ (e.g., Inman, Ryu, and Ward 1975).

The model covariance matrix

I shall adopt a model covariance matrix based on a von Karman covariance function. The general expression for these functions is

$$\Phi_{\nu,L}(z) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{|z|}{L}\right)^{\nu} K_\nu\left(\frac{|z|}{L}\right), \tag{4}$$

Where K_ν is the modified Bessel function of the second kind and order ν, Γ the gamma function, L the maximum correlation length accounted for, and σ controls the amplitude. For $\nu \rightarrow 0$, the von Karman function effectively contains all correlation lengths due to the logarithmic singularity of K_0. This broadband behaviour ensures superior robustness in the inversion, i.e., model structure on all scales will be permitted if required by the data, and it makes the regularisation imposed by the model covariance matrix insensitive to the discretisation (Serban and Jacobsen, 2001; Christensen, Reid, and Halkjær 2009). Maurer, Holliger, and Boerner (1998) demonstrate that the von Karman covariance functions have the two aspects of imposing both smoothing and damping on the inversion problem.

A good approximation to the von Karman functions that allows rapid calculation and analytical integration over model elements can be achieved by stacking single-scale exponential covariance functions with different correlation lengths (Serban and Jacobsen 2001):

$$\Phi_{\nu,L_N}(z) \approx \sigma^2 \sum_{n=0}^{N} k^{n\cdot\nu} \exp\left(-\frac{|z|}{k^n L_N \cdot 0.65}\right) \tag{5}$$

Where L_N is the maximum correlation length represented, k the factor ($k < 1$) between the correlation lengths, N the number of stacked single-scale covariance functions, and σ_ν the standard deviation of the correlation. The factor 0.65 in the exponential denominator is an empirical factor that ensures the fit to the von Karman function. The resulting stacked covariance function is essentially free of correlation scale. The lower and upper limits of the correlation lengths are a mathematical convenience and do not influence the correlation properties at the distance scales typically studied.

In this study, the parameters $v = 0.1$, $C = 0.1$, $L_N = 10{,}000$ km, and $N = 9$ have been used. This means that the covariance function will contain correlation lengths between 6,500 km and 6.5 cm, one per decade. This covers scales of geological variability between the radius of the Earth and small stones; clearly sufficient for most EM data. Notice that the model covariance matrix only depends on the geometry of the multi-layer model; therefore, it needs to be calculated and inverted only once.

The same broadband covariance matrix is used for the lateral correlation as for the vertical smoothness constraint of multi-layer models. Using the broadband covariance matrix with v close to zero is equivalent to an assumption that the variability of the geology is fractal, and using the same order v, of the correlation function for vertical and horizontal regularization means that there are no assumptions that the type of variability of conductivity in the vertical and horizontal directions are different; they can, however, be ascribed different standard deviations σ_v and σ_h. The parameter controlling the strength of the regularisation σ must be given a value that strikes a compromise between smoothness and resolution, and it is chosen pragmatically by inspecting the results of different choices in terms of their geological plausibility.

A STRICTLY HORIZONTAL LATERAL PARAMETER CORRELATION

The fundamental characteristic of the lateral parameter correlation (LPC) method (Christensen and Tølbøll 2009) is that it separates the inversion from the lateral correlation. Three steps are involved in the inversion and lateral correlation of the inversion models:

(i) initial individual inversion of every sounding, e.g., with multi-layer models and vertical smoothness constraints;
(ii) lateral correlation using the LPC method;
(iii) a final individual inversion of data identical to step (i), but with the laterally smooth, correlated models as prior models. The variance of the prior is determined in the LPC procedure in step (ii).

This makes the method much faster than other methods of lateral correlation relying on a simultaneous inversion of a large number of soundings, including lateral constraints in every iterative step. Christensen and Tølbøll (2009) showed through both theoretical and field examples that the LPC method has the desired effect that well-determined parameters have more influence on the correlated models than poorly determined parameters. Since the publication of Christensen and Tølbøll (2009)'s work, the LPC method has been used in the inversion of airborne data from several surveys for both hydrogeophysical and geotechnical purposes (Christensen *et al.* 2009; Christensen and Halkjær 2014; Christensen and Reid 2012; Reid J., Christensen, and Godber; Lawrie et al. 2012a, b).

Having obtained a set of individually inverted models, the correlation is carried out on the model parameters, one at a time. Correlation can be done on layer conductivities or log(resistivities), layer thicknesses, and depths to or elevation of layer boundaries, but, evidently, for multi-layer models only on log(conductivity|resistivity) or linear conductivity.

The values of the selected parameter for all model positions are collected in the parameter vector $\mathbf{p}$. The correlation is formulated as a constrained inversion problem where $\mathbf{p}$ plays the role of the data vector and the model vector to be found $\mathbf{p}_{cor}$ is a smoother version of $\mathbf{p}$. The forward mapping between $\mathbf{p}$ and $\mathbf{p}_{cor}$ is given by

$$\mathbf{p} = \mathbf{I}\mathbf{p}_{cor} + \mathbf{e} \tag{6}$$

Where $\mathbf{I}$ is the identity matrix and $\mathbf{e}$ the observational error. The smoothing is realised by inverting the above relationship incorporating a model covariance matrix $\mathbf{C}_m$, and in the present study, I have used the broadband covariance matrix presented in the previous section. Using the inversion approach of Equation (1) with no prior model included, i.e., $C_{prior} = 0$ in Equation (1), the solution to Equation (5) is:

$$\mathbf{p}_{cor} = \left(\mathbf{I}^T\mathbf{C}_p^{-1}\mathbf{I} + \frac{1}{\sigma_h^2}\mathbf{C}_m^{-1}\right)^{-1}\mathbf{I}^T\mathbf{C}_p^{-1}\mathbf{p} = \left(\mathbf{C}_p^{-1} + \frac{1}{\sigma_h^2}\mathbf{C}_m^{-1}\right)^{-1}\mathbf{C}_p^{-1}\mathbf{p} \tag{7}$$

Where $\mathbf{C}_p$ is a diagonal error covariance matrix of the uncorrelated parameters. Its elements are the variances of the parameters of the uncorrelated models which are identical to the diagonal elements of the posterior covariance matrix of the individual inversions, see equation (2).

The posterior standard deviations of the correlated model parameters are finally estimated as the square root of the diagonal elements of the posterior covariance matrix $\mathbf{C}_{est}$ given as

$$\mathbf{C}_{est}^{cor} = \left(\mathbf{C}_p^{-1} + \frac{1}{\sigma_h^2}\mathbf{C}_m^{-1}\right)^{-1} \tag{8}$$

In the previous version of the LPC method (Christensen and Tølbøll 2009), the model parameter values of a certain layer for the different positions along the profile were involved in the lateral correlation. This meant that the correlation would follow the layer and thereby the topography, and this could have the effect that formations in the correlated model sections would appear to follow the topography. Figure 1(a) illustrates how the model layers in a multi-layer model follow the topography. This is a general problem in lateral correlation of multi-layer models, not only when the LPC method is used but also in the case where inversion and lateral correlation is simultaneous. For inductive EM methods, the adverse effects of this approach would often appear for conductive layers; these are in general better determined than resistive layers, and they can therefore dominate the correlation and extend their presence along the layer, following the topography. Figure 1(b) illustrates how a correlation along the model layers will give preference to formations following the topography. This may be in accordance with the actual geology, but certainly not always;

therefore, a new approach to the correlation needs to be developed. In this new approach—which is the main topic of this paper—for every sounding position and for every layer, the elevation interval of that layer is projected onto all other models, and the parameter is correlated with the average of the parameter values in this elevation interval for all other model positions involved in the correlation. This is illustrated in Figure 1(c). In this way, it is ensured that correlation is strictly horizontal.

The average value of the parameter over an elevation interval is calculated as

$$< p >= (p_1, p_2, p_3, \cdots, p_L) \cdot \mathbf{f} \tag{9}$$

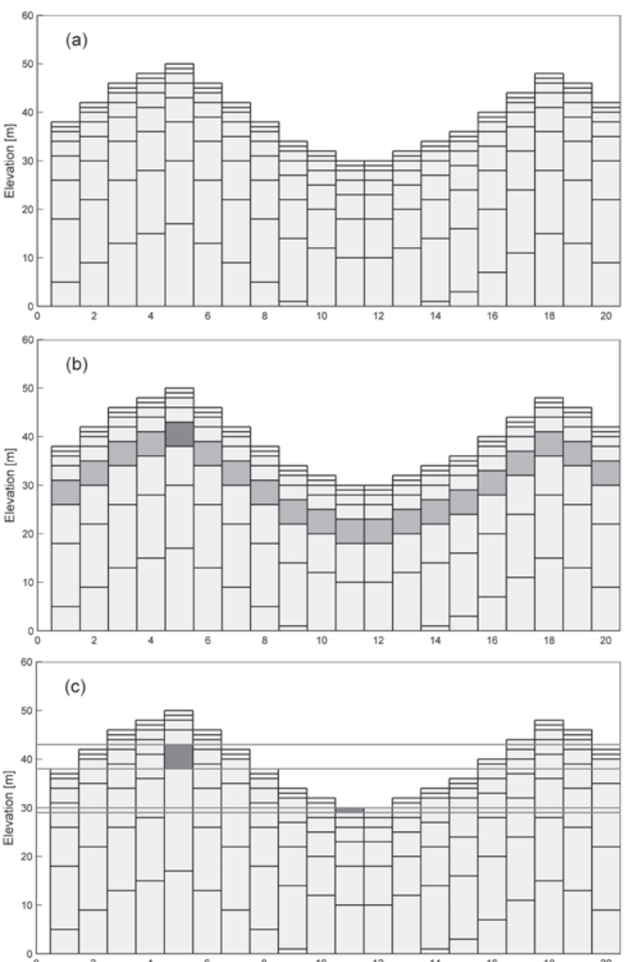

Figure 1 Schematic plots of a model section with topography illustrating the various steps of the horizontal correlation. (a) Model section with topography. (b) Correlation along layers, in this case layer 5, is indicated by the gray layer. (c) Strictly horizontal correlation where the elevation interval of a certain layer (indicated by the dark grey rectangles) is extended to intersect the other models, for each model defining the depth interval of averaging. Note that the depth interval may lie entirely within a layer or be composed of fractions of neighbouring layers. Note also that the models whose surface elevation lie below the lower elevation boundary in question will never be included in the correlation.

Where **f** is the vector containing the fraction of the layers coinciding with the elevation interval and L the number of layers. The variance of the average is calculated as

$$\mathrm{var} < p >= \mathbf{f}^T \mathbf{C}_{est}^{inv} \mathbf{f} \tag{10}$$

Where $\mathbf{C}_{est}^{inv}$ is the posterior covariance matrix of the individual inversions and T indicates transpose.

Once the correlation is done, one by one, for all layers at a certain position, the process is repeated for the next position.

As a consequence of the smoothing involved in the correlation process, the correlated models do not generally fit the data as well as the uncorrelated models. To rectify this, without giving up the smoothness of the correlated models, a final constrained inversion of the data is performed with the correlated values $\mathbf{p}_{cor}$ as both initial and prior model parameters with a diagonal covariance matrix for the prior values defined by the diagonal of $\mathbf{C}_{est}^{cor}$.

The lateral parameter correlation method does not depend on data lying on a straight line or being equidistant because the model covariance matrix is determined only by the actual lateral distance between the models.

Handling large data sets by tiling the plane

An optimal correlation would include all models of a survey in the correlation, but for large surveys with 100,000s or millions of soundings, the inversion problem solved in the LPC procedure becomes intractable, both with regard to storage and computation time. Often, this problem is circumvented by dividing the survey area into partly overlapping subareas that can be handled within a reasonable time and then correlate all subareas (Viezzoli *et al.* 2008, 2009). Some overlap is needed between the subareas to avoid edge effects, but even if such overlap is implemented, the information from soundings outside of the area is not involved in the correlation. This means that the long-wavelength behaviour of the correlated parameters will not be able to influence the correlation, although ideally it should.

In this study, a different approach is taken that at the same time ensures that information from all models is included in the correlation and that the inversion problem is of a tractable size. The dilemma can be solved by observing that the correlation effect of parameters at long distances from the central sounding is transferred primarily through averages, i.e., a group of models, binned and averaged into an average model at an average position, will exert their influence on the correlation in much the same way as if they had been included individually. By binning models within subareas of increasing size with distance, it is possible to make all the information in the survey area contribute and still keep the computations involved in the correlation at a reasonable level.

In the approach taken here, the principle of subdivision is taken to its radical extreme: each subdivision consists of one central circular cell with a radius of one half of a typical distance unit, containing only one sounding position surrounded by a circular

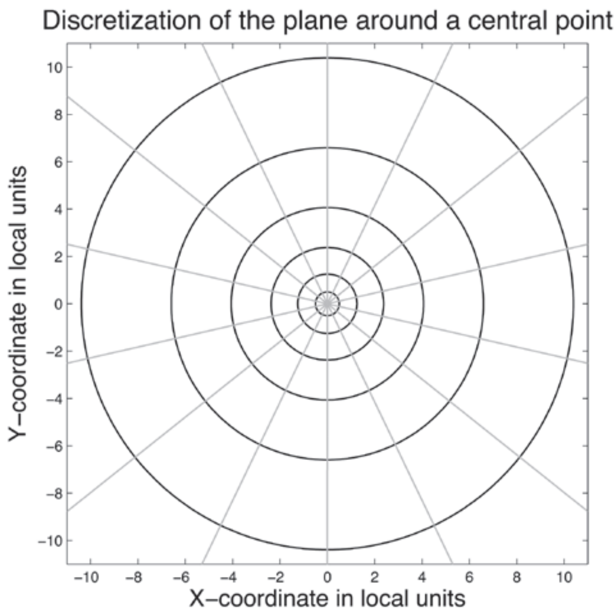

Figure 2 The discretisation of the plane around a central point. The radius of the central cell is 0.5 times the local distance unit, the exponential factor is 1.5, and there are 14 azimuthal subdivisions.

division of increasing radii. Each circular annulus is divided into N sections each extending over an angle of $2\pi/N$ (see Figure 2). In this approach, there are no assumptions regarding the organisation of the data positions in terms of being oriented linearly or any other way.

The increasing size of the subareas with distance from the central cell is implemented by selecting the distance between dividing circles to increase exponentially, i.e., the thickness of any annulus is a factor times the previous one. The distance unit is chosen as the typical distance between soundings in the vicinity of the central sounding. Because the distance unit is defined locally, the discretisation is capable of adapting to changes in the density of models within the survey area. In the field example, I have chosen the exponential factor to be 1.5. Within a circular annulus, the full circle is then divided into a number of subsections chosen so that the subareas have approximately the same linear extent in the radial and azimuthal directions.

Input to the averaging within the bins of the tessellation defined above can of course be restricted to soundings along a flight line only; therefore, the formulation covers both 1D and 2D correlation. In 1D correlation, only the points on a profile line are averaged in padding cells. In 2D correlation, all points in the plane are included in the averages over the padding cells.

The practical implementation is the following:

For every position:

- Define the tessellation.
- For every layer of the central sounding:
  > Find the average parameter and its variance for all other soundings in the elevation interval defined by the layer of the central sounding.
  > Form the average over the ensemble of soundings within the cells of the tessellation of the plane and calculate the variance of the average.
  Within each cell, the lateral position of the average model is the simple mean of the lateral position of the composite soundings.
  > Calculate the model covariance matrix relating to the position of the central sounding and the positions of the cell averages in all occupied cells.
  > Form the diagonal data error covariance matrix.
  > Solve the inversion problem and store the value of the correlated layer parameter and its posterior variance.
- Go to the next layer of the central sounding.

Go to the next position.

It is clear that substantial amounts of computation is used in the strictly horizontal approach to lateral correlation because a new tessellation must be done for every new sounding position and the averaging over elevation intervals, binning and averaging within the cells, calculation of matrices, and the solution to the inversion problem must be done for every layer of every sounding of the whole survey. The computation time can be reduced by limiting the correlation to soundings within a circle defining a maximum distance of correlation. It is quite reasonable to exclude soundings further away than a maximum distance as long as the maximum distance allows the full volume of the sensitivity function of the measurement with the largest lateral extent to be included. It is also geologically reasonable to exclude soundings further away than typical length scales of the landscape elements of the survey area.

Correlation of models from multiple methods

The LPC method correlates models; therefore, it is of no importance where the models come from, i.e., they can be inversion results from different geophysical methods, e.g., airborne EM, ground resistivity, or borehole logs. All that is required is that the model parameters come with an estimate of their uncertainty. This makes the LPC method quite flexible for correlating all geophysical information within an area. The strictly lateral correlation procedure also possesses the flexibility that models with a varying number of layers can be correlated. This is not possible if the correlation is conducted along layers.

Parallelisation

The initial inversion, the lateral correlation and the final inversion can be divided into subgroups and distributed over as many computing nodes as are available. The initial and final inversions are single-site inversions and do not depend on communication with other nodes. The correlation can also be subdivided and distributed on any number of computing nodes. No communication is needed during the calculations as long as all nodes have reading access to the entire set of inversion models from the initial inversion. All three steps of the laterally correlated inversion procedure are thus ‘embarrassingly parallel’ computational problems.

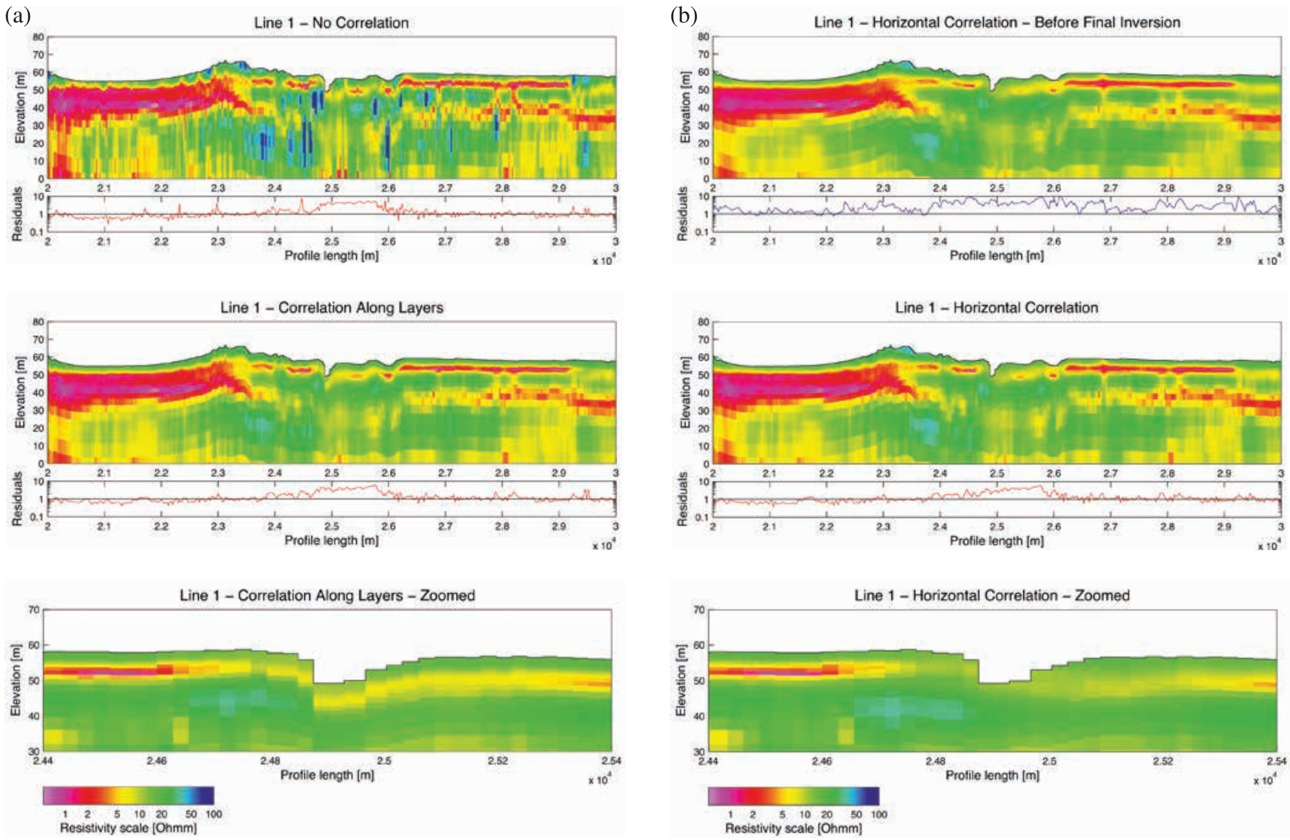

Figure 3 (a) Model sections from a 10-km section of line 1 (arbitrary numbering) from the BHMAR survey. Top frame: The model section after the first individual, uncorrelated inversion. Middle frame: The model section after the first individual inversion, a lateral correlation along the layers, and the final inversion. Bottom frame: A 1-km zoomed version of the middle frame highlighting the important feature. (b) Model sections from a 10-km section of line 1 (arbitrary numbering) from the BHMAR survey. Top frame: The model section after the first individual, uncorrelated inversion followed by the strictly horizontal lateral correlation, but before the final inversion. Middle frame: The model section after the first individual inversion, a strictly horizontal lateral correlation, and the final inversion. Bottom frame: A 1-km zoomed version of the middle frame highlighting the important feature.

A FIELD EXAMPLE: THE BROKEN HILL SURVEY

In the paper by Christensen and Tølbøll (2009), a theoretical/numerical example is shown as proof of concept for the basic approach of the LPC method. In this paper, I shall therefore go directly to the field examples that illustrate the advantages of the strictly horizontal correlation.

An inversion example from the BHMAR data set

I will compare the results of using a lateral correlation along layers and a strictly horizontal correlation using data from two lines of the helicopter-borne transient data from the Broken Hill Managed Aquifer Recharge (BHMAR) project. A total of 30,000 line km of data were acquired in 2008 with the SkyTEM system (Sørensen and Auken 2004) in a standard duel-moment mode with gate centre times between 16 and 895 µs for the low moment and between 85 µs and 8.84 ms for the high moment,

The aim of the BHMAR project was to map fresh and brackish ground water resources in the Broken Hill area and to point out locations for managed aquifer recharge experiments (Lawrie *et al.* 2012a,b). By implementing managed aquifer recharge, water from annual/biannual large precipitation events would not be lost to evaporation but stored underground for use between the events whereby more water could be left in the Darling–Murray river systems to improve the reliability of domestic and industrial/agricultural water supply and the health of the rivers and the general environment. In addition to the AEM data, the BHMAR project involved borehole induction log data, lithological sampling, hydraulic modelling, assessment of flora and fauna and an extensive geological interpretation including the effects of neotectonics (Lawrie *et al.* 2012a).

In this field example, illustrating the difference between a lateral correlation along model layers and a strictly horizontal lateral correlation, helicopter-borne transient data are used. The initial inversion is done with a multi-layer model with 30 layers and with a top layer thickness of 0.5 m and a depth to the top of the bottom layer of 200 m. The initial model is a homogeneous half-space with a resistivity of 20 Ωm, and inversion is done with no prior model, i.e., $C_{prior} = 0$ in equation (1).

The basic distance unit—the average distance between soundings—was 25m, the factor for the exponential sequence of radii of the padding cells was set to 1.5 corresponding to ~6 radii per decade, and correlation was done on log(conductivity) with a standard deviation of the model covariance matrix of $\sigma_h = 0.7$.

Two field examples are shown in Figures 3(a) and (b), and in Figures 4(a) and (b) for 'Line 1' and 'Line 2', respectively (the exact locations of the two lines is still proprietary information). Figures 3(a) and (b) show the consequences of a correlation along layers and a strictly horizontal correlation for a thin near-surface well-conducting layer, geologically interpreted as a clay layer, around erosion structures of a river valley. In Figure 3(a), in the top frame, the model section after the initial uncorrelated inversion is shown together with the data residual of the inversion. In the middle frame, the result after correlation along layers followed by the final inversion is shown with residuals, and the bottom frame is a zoom-in on the most interesting part of the section around the river valley. In Figure 3(b), the top frame shows the model section after a strictly horizontal correlation but before the final inversion, together with the residual of the correlated models. The middle frame of Figure 3(b) shows the models after a strictly horizontal correlation and a final inversion with residuals, whereas the bottom frame is a zoom-in on the river valley.

It is seen that where the conductivity structures are predominantly horizontal, there is no difference between correlating along layers and strictly horizontally. However, in the central part of the model section, a river has eroded the landscape, and it is clear that the correlation along layers 'recreates' the near-surface well conducting layer below the river, when in fact this layer is eroded away. The strictly horizontal correlation shows that the layer stops at the river and continues on the other side of the river valley, which evidently is geologically correct.

Comparing the data residuals of the various inversions, it is seen that they are approximately the same for both the uncorrelated models and for the correlated models, both for correlation along layers and for the strictly horizontal correlation. Comparing the model sections after strictly horizontal correlation, but before and after the final inversion (Figure 3b, top and middle frame), it is seen that the two model sections are very much alike. The data residual of the horizontally correlated section before the final inversion is a factor of 2–3 higher than the residual after the final

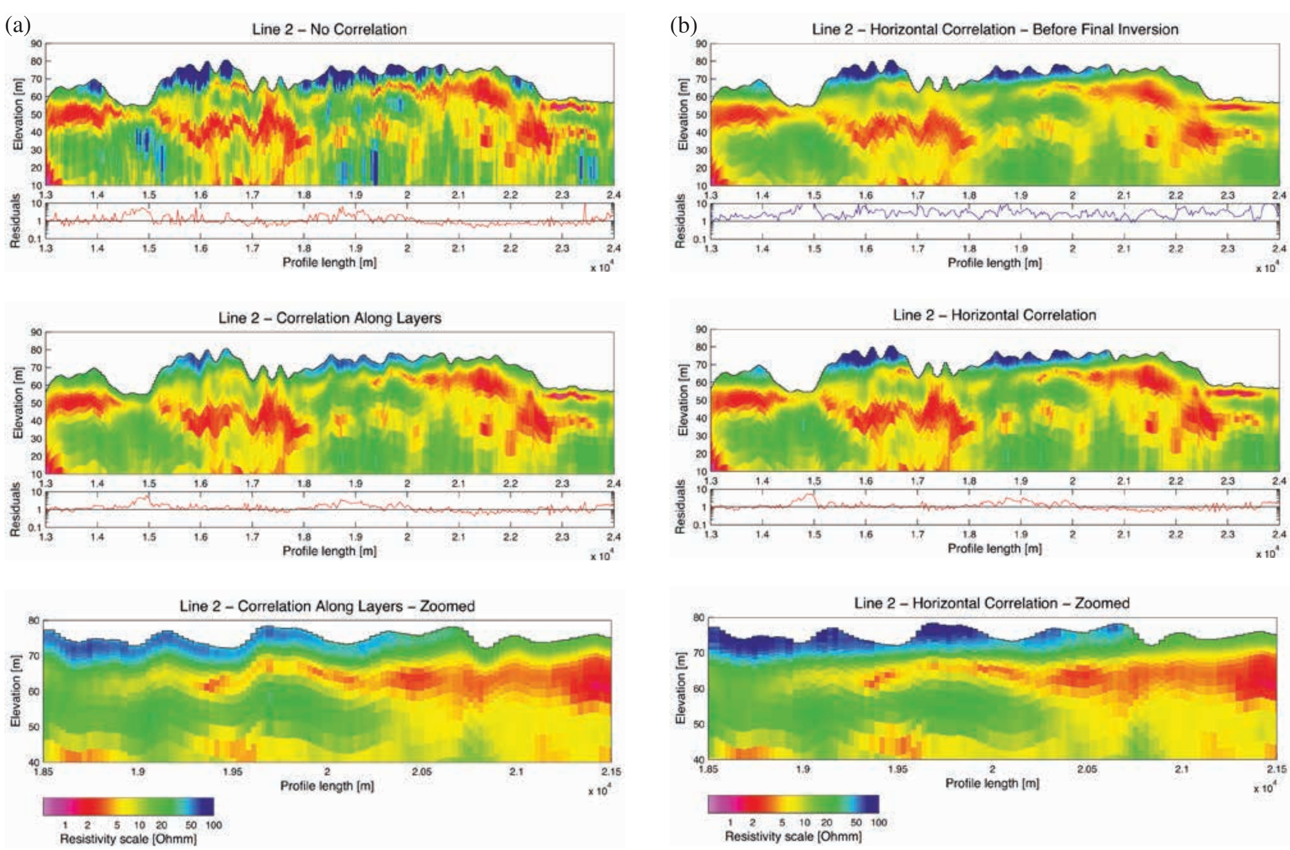

Figure 4 (a) Model sections from a 10-km section of line 2 (arbitrary numbering) from the BHMAR survey. Top frame: The model section after the first individual, uncorrelated inversion. Middle frame: The model section after the first individual inversion, a lateral correlation along the layers, and the final inversion. Bottom frame: A 1-km zoomed version of the middle frame highlighting the important feature. (b) Model sections from a 10-km section of line 2 (arbitrary numbering) from the BHMAR survey. Top frame: The model section after the first individual, uncorrelated inversion followed by the strictly horizontal lateral correlation, but before the final inversion. Middle frame: The model section after the first individual inversion, a strictly horizontal lateral correlation, and the final inversion. Bottom frame: A 1-km zoomed version of the middle frame highlighting the important feature.

inversion; the final inversion only modifies the model section slightly.

Figures 4(a) and (b) follows a layout similar to Figures 3(a) and (b). In this example, the different outcomes of correlation along layers and horizontally are investigated for low-conductivity near-surface depositional structures, geologically and geomorphologically interpreted as sand dunes.

Again, it is seen that where conductivity structures are predominantly horizontal, there is no difference between correlating along layers and strictly horizontally. However, in the parts of the model sections where there is high topographical variability (the sand dunes), it is seen that the correlation along layers pulls up the more conductive structures below the sand dunes to be parallel to the surface, wheras the strictly horizontal correlation reveals that the sand dunes are deposition structures unrelated to the sedimentation of the topmost well conducting clay layer.

Computation time for the strictly horizontal correlation, for example, 6,000 soundings on a line, is 227 s, 70% of which is used on the averaging and binning and 30% on solving the inversion problem. For correlation along layers, the computation time is 123 s, 47% of which is used on the averaging and binning and 53% on solving the inversion problem. This illustrates that the advantage of a strictly horizontal correlation comes at the price of an increased computation time due to the many calls to the averaging routines.

CONCLUSIONS

The basic characteristic of the LPC methodology, i.e., the inversion and the lateral correlation are separated, can be seen as the first three steps in an infinite iterative approach of inversion and correlation. Numerical experiments have shown (Christensen and Tølbøll 2009) that nothing is achieved by repeating the process any more that the first three steps outlined in this paper. The separation of inversion and correlation offers some advantages over the traditional approach of integrating the correlation in every iterative inversion step: it is faster and it is very flexible with regard to including inversion models from different sources, among these borehole data. The method is furthermore free of any restrictions with respect to the organisation of the model positions, and in the strictly lateral approach also with respect to the number of layers of each inversion model. The LPC method can be used for any dimensionality of the correlation, e.g., on a line (1D), in a plane (2D), or in a 3D volume.

The broadband covariance matrix that inherently includes all (relevant) correlation lengths does not require the user to postulate distance dependence of the correlation. Only one parameter, the strength of the correlation, is needed. For large datasets, a tessellation of the plane into averaging cells of increasing size with distance from the central sounding permits all relevant information to be included in the correlation by dividing the inversion problem of the correlation into many small problems that are easily solved.

In areas with significant topography and areas where erosion processes have formed the landscape, the strictly horizontal correlation outperforms traditional correlation along layers and avoids artefacts common in the correlation along layers. Strictly horizontal correlation is computationally more demanding than correlation along layers, but considering the advantages, the increase is manageable, and the LPC method lends itself easily to ‘embarrassing parallel’ computations with no need of data transfer between nodes.

In this paper, the LPC methodology has been applied to EM data, but the approach can easily be adapted to correlation in connection with inversion problems of other data types.

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