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Joint inversion of aquifer test, MRS, and TEM data

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Abstract This paper presents two methods for joint inversion of aquifer test data, magnetic resonance sounding (MRS) data, and transient electromagnetic data acquired from a multilayer hydrogeological system. The link between the MRS model and the groundwater model is created by tying hydraulic conductivities (*k*) derived from MRS parameters to those of the groundwater model. Method 1 applies *k* estimated from MRS directly in the groundwater model, during the inversion. Method 2 on the other hand uses the petrophysical relation as a regularization constraint that only enforces *k* estimated for the groundwater model to be equal to MRS derived *k* to the extent that data can be fitted. Both methodologies can jointly calibrate parameters pertaining to the individual models as well as a parameter pertaining to the petrophysical relation. This allows the petrophysical relation to adapt to the local conditions during the inversion. The methods are tested using a synthetic data set as well as a field data set. In combination, the two case studies show that the joint methods can constrain the inversion to achieve estimates of *k*, decay times, and water contents for a leaky confined aquifer system. We show that the geophysical data can assist in determining otherwise insensitive *k*, and vice versa. Based on our experiments and results, we mainly advocate the future application of method 2 since this seems to produce the most reliable results, has a faster inversion runtime, and is applicable also for linking *k* of 3-D groundwater flow models to multiple MRS soundings.

1. Introduction

Flow of groundwater in the subsurface is governed by the spatial distribution of hydraulic conductivity. Traditionally, estimates of hydraulic conductivity are acquired through analysis of aquifer test or slug test data. However, such tests require admittance to wells that usually are scarce and expensive to make. Furthermore, in most cases, an aquifer test can only be used to estimate the transmissivity or the leakance of layers while variations in hydraulic conductivity within a layer rarely can be resolved. Some geophysical methods have the potential to allow estimation of the hydraulic conductivity at locations without wells and at depths not reached by wells, but only to the extent that the geophysical measurements can be paired with one or more hydraulic tests that provide information about the relation between geophysical parameters and hydraulic conductivity.

Mazác et al. [1985] reviewed literature that tries to establish relations between hydraulic conductivity and electrical resistivity of the saturated subsurface. The relations are based on variations of Archie's law stating that the resistivity of a fluid-saturated nonconductive rock matrix is the product of the formation factor and the resistivity of the pore water. The formation factor is the ratio of the tortuosity factor and the porosity raised by the exponent of cementation. For a fluid-saturated nonconductive porous rock with constant fluid resistivity, measurements of bulk electrical resistivity can be used to map variations in bulk porosity (where "bulk" is the volume of the measurement) within a larger rock volume [*Kirsch*, 2006, pp. 8–9]. Hydraulic conductivity depends on porosity, so electrical resistivity also has the potential to be informative about hydraulic conductivity. However, as discussed by, e.g., *Bear* [1979, pp. 66–69] and *Kirsch and Yaramanci* [2006, pp. 440–443] hydraulic conductivity also depends on other factors such as pore size distribution, shapes of grains, tortuosity, and specific surface area. Existence of a simple relation between electrical resistivity and hydraulic conductivity is also weakened when the soil matrix itself to some extent is an electrical conductor, for example, when it contains clay minerals. Such a relation may therefore hardly exists (or only exist as a crude approximation) for environments with spatially highly variable clay contents: this could, for example, be glacial environments containing both tills with varying clay content and outwash deposits with little or no clay content. Estimation of hydraulic conductivity from electrical resistivity is, therefore, often done using

a site-specific empirical linear log-log relation between the two [*Mazác et al*., 1985; *Slater*, 2007]. Some studies have proposed and used such a petrophysical relation to incorporate geophysical data directly in groundwater model calibration [*Dam and Christensen*, 2003; *Hinnell et al*., 2010]. *Dam and Christensen* [2003] used inversion to estimate not only the hydraulic conductivity field, but also two parameters of the petrophysical relation between hydraulic conductivity and electrical resistivity.

Magnetic resonance sounding (MRS) is a geophysical method that has potential for estimation of hydrological parameters. The initial amplitude of the MRS signal is directly proportional to the free water content of the subsurface, and the decay of this signal can be related to the pore structure in the water saturated sediment [*Legchenko et al*., 2002; *Vouillamoz et al*., 2007]. The parameters estimated from MRS, therefore, appear to be rather directly related to the hydraulic properties of the subsurface such as hydraulic conductivity or transmissivity [*Lachassagne et al*., 2005; *Legchenko et al*., 2002; *Lubczynski and Roy*, 2003; *Plata and Rubio*, 2008], as well as effective porosity and specific yield [*Boucher et al*., 2009; *Lachassagne et al*., 2005; *Lubczynski and Roy*, 2003]. However, an empirical site-specific petrophysical relation is also required to estimate hydraulic conductivity from the MRS estimated water content and decay time. To provide reliable estimates of hydraulic conductivity, the parameters of the petrophysical relation should be estimated from local pumping tests and MRS data; the importance of this was demonstrated by, e.g., *Legchenko et al*. [2002]. Traditionally, the empirical relation is established by first doing independent inversion (as defined by *Ferré et al*. [2009]) of the data from locally conducted aquifer tests, MRS soundings, and TEM soundings to estimate the respective hydraulic and geophysical parameters, and then by using these estimates to sequentially estimate (a small number of) petrophysical parameters that characterize the relation between the hydraulic and geophysical parameters.

In this study, we propose two methods for full joint hydrogeophysical inversion (as defined by *Ferré et al*. [2009]) of MRS, transient electromagnetic (TEM) and aquifer test data collected in a confined aquifer system. The inversion schemes apply an empirical petrophysical relation between water content and decay time estimated by MRS and hydraulic conductivity. We use the alternative relations proposed by *Seevers* [1966] and *Kenyon et al*. [1988], respectively. The parameters estimated by the joint inversion are thicknesses, hydraulic conductivities, storage coefficients, electrical resistivities, MRS water contents, and decay times of a multilayer hydrogeological system, as well as a parameter of the petrophysical relation between hydraulic conductivity and MRS water content and decay time. The two joint inversion methods use the petrophysical relation differently, which give them each their advantages and disadvantages when it comes to practical application. The TEM data set is included in the joint inversion since *Braun and Yaramanci* [2008] and *Behroozmand et al*. [2012a] found that a well-calibrated resistivity structure is important for a successful MRS interpretation, especially in presence of a conductive ground.

The main objective of the paper is to present and demonstrate the two joint inversion methods, to discuss their advantages and disadvantages for practical application, and their advantages compared to independent inversions. For the demonstration, we use both a synthetic case of a 40 m deep five-layer system with two aquifers and three semipermeable layers, and a field case where the data set was collected by the USGS in the Central Platte Natural District, Nebraska [*Irons et al*., 2012].

The following begins with a section that briefly describes the aquifer test, MRS, and TEM methods and their traditional use. It also describes the two new joint hydrogeophysical inversion methods for the fully integrated data analysis and parameter estimation. The synthetic and field test cases and results are presented in the following two sections, respectively. Then the methods and results are discussed, while the final section draws the conclusions.

2. Methods

2.1. Aquifer Test Analysis

Aquifer testing has been used for decades to estimate hydraulic parameters of aquifers and aquitards. A test is typically conducted by pumping groundwater at constant rate from a well that screens an aquifer while observing the resulting drawdown of hydraulic head in observation wells that screen the aquifer and/or surrounding formations. The drawdown data are analyzed using a groundwater model that can simulate the same type of system: the values of the hydraulic parameters of the groundwater model are calibrated to make the simulated drawdown responses fit the observations made during the test. The parameter

values that can be estimated are typically the transmissivity and storativity of the pumped aquifer, the leakance of leaky aquitards that separate the pumped aquifer from shallower and/or deeper aquifers, and sometimes also the transmissivity and storativity of the shallower or deeper aquifer(s). The hydraulic conductivity and the specific storage of aquifers and aquitards can only be estimated in specially designed aquifer test cases, or when layer thicknesses can be estimated independently through, e.g., deep borehole information.

Aquifer tests of relatively simple groundwater systems can be simulated and analyzed using analytical models whereas more complex systems require use of a numerical groundwater model. Here we study testing in a layered system with multiple aquifers and aquitards where the pumping causes drawdown not only in the pumped aquifer but in all layers of the system. The simulation is done using a version of MODFLOW-2000 [*Harbaugh et al*., 2000] that *Clemo* [2002] modified to ease use of cylindrical grid geometry.

2.2. The MRS Method

Magnetic resonance sounding (MRS), also called surface-NMR (where NMR stands for Nuclear Magnetic Resonance), is a noninvasive geophysical technique, which has emerged as a promising surface-based geophysical technique for characterization of groundwater resources [e.g., *Plata and Rubio*, 2007]. Based on the physical principle of NMR, a large volume of protons of the hydrogen nuclei is excited within the earth's static magnetic field. The excitation is done by passing an alternating current, tuned at local, the Larmor frequency, through a large transmitter loop laid out on the surface. After the current is switched off, the decaying NMR signal, called "Free Induction Decay" (FID) is measured inductively in a receiver loop on the surface. The initial amplitude of this signal is directly proportional to the water content [e.g., *Legchenko et al*., 2004; *Legchenko et al*., 2002], the decay time contains information of the pore structure [e.g., *Legchenko and Valla*, 2002; *Vouillamoz et al*., 2007], and in combination they can potentially be used for hydraulic conductivity estimation [e.g., *Legchenko et al*., 2002]. A series of measurements at increasing pulse moments, which is the product of current amplitude and pulse duration $(q = I_0\tau)$, provides depth information.

In the following, 1-D MRS forward responses are modeled applying the method of *Behroozmand et al*. [2012b], which is incorporated in the AarhusInv inversion package (E. Auken et al., EM1DINV part A: A highly versatile and robust inversion code implementing accurate system forward modeling and arbitrary model constraints, submitted to *Exploration Geophysics*, 2013). In this study, a monoexponential MRS signal is assumed in each layer, corresponding to a homogeneous-layered structure. Hencê, the MRS forward response is given by

$$V(q,t) = \int K(q,z,\rho)w(z)exp\left(-\frac{t}{T_2^*(z)}\right)\,dz \tag{1}$$

in which $V(q,t)$ is the measured signal current induced in the receiver coil; $K(q,z,\rho(z))$ is the 1-D MRS kernel depending on the pulse moment $[q]$, the depth $[z]$, and the resistivity structure $[\rho(z)]$; $w(z)$ and $T_2^*(z)$ denote depth-specific water content and decay time; and t is the measurement time. The resistivity structure needs to be included in the kernel since it determines the excitation magnetic field values in the subsurface. Building the MRS-model on a flawed resistivity structure might introduce considerable errors in the MRS forward response [*Behroozmand et al*., 2012a].

In the following, we use block inversion to estimate the thickness, the water content $[w]$, and the FID decay time $\left[T_2^*\right]$ of each layer in the hydrogeological system on basis of $V(q,t)$ measurements, i.e., so-called QT inversion of the data [*Behroozmand et al*., 2012b; *Müller-Petke and Yaramanci*, 2010]. In this context, "block" just means that the MRS model is set up (parameterized) to simulate the hydrogeological system as consisting of a small number of homogeneous layers.

2.3. Deriving Hydraulic Conductivity From MRS

Several studies have shown that the NMR-estimated water content and decay time can be linked to hydraulic conductivity through the Schlumberger-Doll Research (SDR) equation whose original formulation is given as:

$$k_{SDR} = C_p w^a (T_2)^2 \tag{2}$$

where k_{SDR} is the hydraulic conductivity of the sample, w is the corresponding water content, and T_2 the corresponding decay time, C_p and a are empirical parameters [e.g., *Legchenko et al*., 2002; *Lubczynski and Roy*, 2003; *Mohnke and Yaramanci*, 2008]. Here T_2 represents the transversal decay time. Under ideal conditions, namely a uniform magnetic field, $T_2 = T_2^*$ [e.g., *Grunewald and Knight*, 2011; *Mohnke and Yaramanci*, 2008], and the relaxation rate (T_2^{-1}) of a single pore can be subdivided into two terms; the bulk water relaxation rate (T_{2B}^{-1}) and the surface relaxation rate (T_{2S}^{-1}). Under the assumption of fast diffusion regime and no/limited pore coupling, the surface relaxation rate in a single pore is given by $T_{2S}^{-1} = \rho_2 \frac{S}{V}$, where ρ_2 is the surface relaxivity, and $\left(\frac{S}{V}\right)$ is the surface area to volume ratio of the pore [*Brownstein and Tarr*, 1979; *Dunn et al*., 1999]. Moreover, it can be assumed that $T_{2S}^{-1} \gg T_{2B}^{-1}$, meaning that $T_2^{-1} \approx T_{2S}^{-1}$. Under these restrictions, the mean value of the T_2 (and thereby T_2^*) distribution can be linked to the pore geometry if ρ_2 is constant throughout the investigated deposits. The assumption of a uniform magnetic field is however most often violated to some extent and the FID relaxation rate is given by $T_2^{*-1} = T_2^{-1} + T_{2IH}^{-1}$, where T_{2IH}^{-1} represents the dephasing rate caused by the inhomogeneous magnetic field and the influence of molecular diffusion [e.g., *Grunewald and Knight*, 2011; *Müller et al*., 2005]. In the cases where T_{2IH}^{-1} dominates over T_{2S}^{-1}, the estimate of T_2^* provides little or no information about the subsurface pore structure. Inhomogeneity in the magnetic field is often created due to the presence of paramagnetic minerals such as iron or manganese [e.g., *Müller et al*., 2005] and/or large-scale variations in the earth's field (in MRS applications). In order to estimate hydraulic conductivity from T_2^* derived from inversion of the MRS data set it must, therefore, be assumed that the contribution from magnetic field inhomogeneity can be neglected. By acknowledging this limitation, the hydraulic conductivity derived from MRS in present study has been renamed to k_{MRS} and the equation applied takes the form:

$$k_{MRS} = C_p w^a (T_2^*)^2 \tag{3}$$

In order to estimate hydraulic conductivity from MRS, equation (3) has to be calibrated to estimates of k obtained in the vicinity of the MRS sounding. Such information is often taken from aquifer tests [e.g., *Legchenko et al*., 2002; *Nielsen et al*., 2011], which can provide reliable estimates of aquifer transmissivity (depth integration of hydraulic conductivity), and under special conditions hydraulic conductivity and leakage of aquitards. Depending on the expected ratio between pore sizes and pore throats, a is often set to 1 as derived by *Seevers* [1966], or to 4 as suggested by *Kenyon et al*. [1988]. In the following, the version of (3) using a = 1 will be referred to as Seevers relation and using a = 4 will be referred to as Kenyons relation. Based on estimates of transmissivity or hydraulic conductivity, the parameter C_p can be determined by regression analysis between "transmissivity divided by thickness" (an estimate of k) and corresponding values of the regressor "$w^a * (T_2^*)^2$" c.f. the empirical relation (3). In the following, C_p and other parameters will be estimated simultaneously by joint inversion, while keeping the exponents a fixed to either 1 or 4.

2.4. TEM Sounding

The TEM method is a time-domain electromagnetic method commonly used in hydrogeological investigations [e.g., *Danielsen et al*., 2003; *Jørgensen et al*., 2003; *Sandersen and Jørgensen*, 2003]. In ground-based applications, the method uses a transmitter loop and a receiver loop placed on the ground. A direct current is passed through the transmitter loop, which results in a static primary magnetic field. When the current is switched off, the primary field creates eddy currents, which are transmitted downward and outward in the subsurface. This current introduces a secondary magnetic field, which can be measured on the surface using the receiver loop. With time, the resistivity structure of the subsurface will weaken the currents induced by the primary field. The early secondary responses, which are measured in the receiver coil, will contain information of the near-surface structures, while responses of deeper structures can be measured as the current moves downward [*Christiansen et al*., 2006, pp. 179–191]. Since the decay of the induced currents is slowest in low-resistivity layers, the method is most sensitive to this type of formations.

In the present study, TEM data are used in the joint inversion to help estimation of the thickness and electrical resistivity of the hydrogeological layers. The TEM forward responses are simulated using AarhusInv (E. Auken et al., submitted manuscript, 2013).

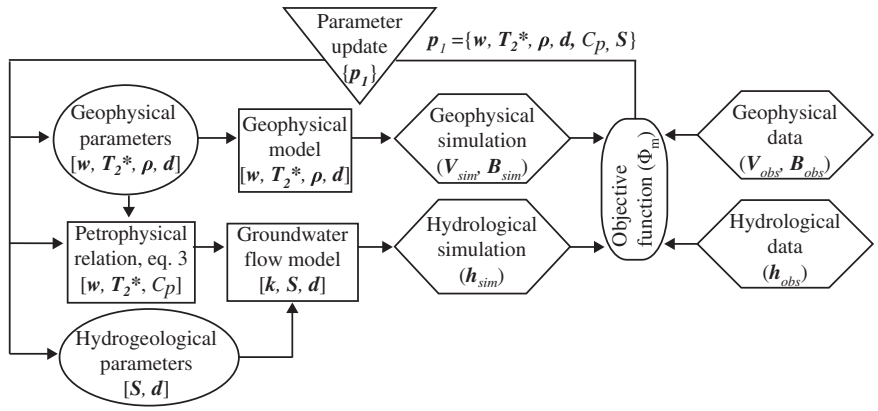

Figure 1. Flowchart for the inversion scheme of method 1. Estimated parameters are enclosed in square brackets, data and simulated equivalents are in rounded brackets, and the parameter update vector is shown in curly brackets.

2.5. Parameter Estimation by Joint Inversion

Traditionally, TEM, MRS, and aquifer test analyses are conducted independently. TEM measurements are used to estimate the electrical resistivity structure of the subsurface. These estimates are used to compute the Kernel during inversion of MRS data that estimate the water contents and decay times of inversion layers. Aquifer test analysis is conducted to estimate hydraulic parameters such as transmissivity and storativity. The transmissivity estimates are finally used with the MRS parameter estimates to determine "correlation parameter(s)" (e.g., C_p in equation (3)) between hydraulic conductivity and MRS parameters.

Previously, *Behroozmand et al.* [2012a] demonstrated that it is beneficial to make joint inversion of TEM and MRS data to determine the respective parameter values of the two models. Here we will describe two joint inversion methods that can be used to simultaneously estimate all geophysical and hydraulic parameters of a layered hydrogeological system (bold symbols indicate vectors): the thickness [$\boldsymbol{d}$], the electrical resistivity [$\boldsymbol{\rho}$], the water content [$\boldsymbol{w}$], the decay time [$\boldsymbol{T}_2^*$], the hydraulic conductivity [$\boldsymbol{k}$], and the specific storage [$\boldsymbol{S}$] of each layer, plus the C_p parameter of relation (3) that is necessary to calculate a hydraulic conductivity estimate from MRS parameters.

Both joint inversion methods quantify the misfit of model simulated responses to the data by the measurement objective function

$$\Phi_{\mathrm{m}} = n_{pt}^{-1}\sum_{i=1}^{n_{pt}}\left(\frac{h_{obs,i}-h_{sim,i}}{\sigma_{pt,i}}\right)^2 + n_{mrs}^{-1}\sum_{i=1}^{n_{mrs}}\left(\frac{V_{obs,i}-V_{sim,i}}{\sigma_{mrs,i}}\right)^2 + n_{tem}^{-1}\sum_{i=1}^{n_{mrs}}\left(\frac{B_{obs,i}-B_{sim,i}}{\sigma_{tem,i}}\right)^2 \tag{4}$$

where n_{pt}, n_{mrs}, and n_{tem} are the number of observations from the aquifer test, the MRS sounding, and the TEM sounding, respectively; h_{obs} is hydraulic head observed during the aquifer test and h_{sim} its model simulated equivalent; V_{obs} and V_{sim} are observed and corresponding simulated MRS measured signal in the receiver coil; B_{obs} and B_{sim} are observed and simulated decay data from TEM; while σ_{pt}, σ_{mrs}, and σ_{tem} are the noise levels (standard deviation) for the head, MRS, and TEM data, respectively. Each of the three terms on the right-hand side of (4) is normalized by the inverse of the corresponding number of observations to promote a balanced weight between the three sets of measurement data.

Method 1 estimates the values of the reduced parameter vector $\boldsymbol{p}_1 = \{\boldsymbol{d}, \boldsymbol{\rho}, \boldsymbol{w}, \boldsymbol{T}_2^*, \boldsymbol{S}, C_p\}$ by minimizing the measurement objective function (4). The hydraulic conductivities $\boldsymbol{k}$ are not estimated directly but are computed throughout the inversion process from relation (3) using the estimated values of $\boldsymbol{w}$, $\boldsymbol{T}_2^*$, C_p, and a known (or assumed) value for a. Sensitivity analysis of both the synthetic and the field data set show that C_p and a cannot be estimated independently on basis of the studied types of data due to parameter correlation, neither by method 1 nor by method 2. The flowchart of method 1 is illustrated in Figure 1.

Since hydraulic conductivities are calculated directly from $\boldsymbol{w}$ and $\boldsymbol{T}_2^*$, the link between the groundwater model and the geophysical models is fixed: changes in geophysical parameters result directly in change of the simulated hydraulic head values. In the present case, parameter sensitivities are calculated by using

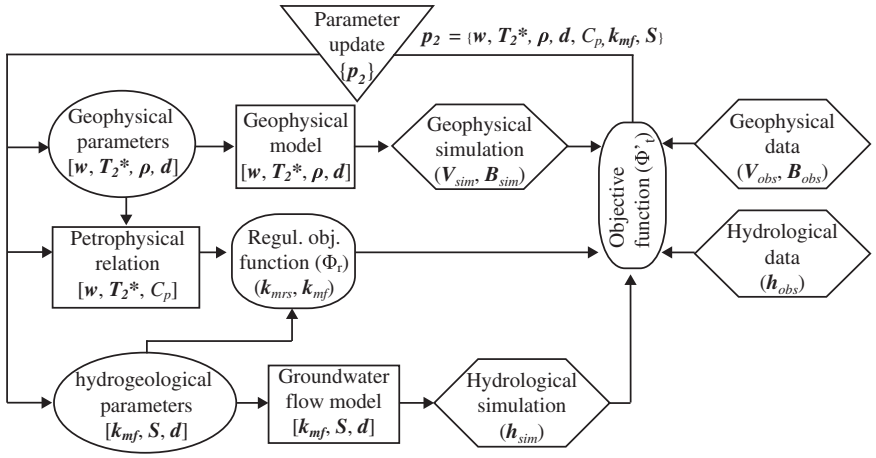

Figure 2. Flowchart for the inversion scheme of method 2. Estimated parameters are enclosed in square brackets, data and simulated equivalents are in rounded brackets, and the parameter update vector is shown in curly brackets. In the regularization objective function (Φ_r), the hydraulic conductivities estimated from MRS (k_{mrs}, *calculated by eq.* 3) are used as "observations" for the hydraulic conductivities used in the groundwater model (k_{mf}).

perturbation methods, and due to the fixed link both the geophysical models and the groundwater model must be run to calculate each column of the sensitivity (Jacobian) matrix.

Method 2 estimates the values of the full parameter vector $\boldsymbol{p}_2=\{\boldsymbol{d},\boldsymbol{\rho},\boldsymbol{w},\boldsymbol{T}_2^*,\boldsymbol{S},\boldsymbol{k_{mf}},C_p\}$ by minimizing the regularized objective function:

$$\Phi_t=\Phi_m+\mu\Phi_r \tag{5}$$

where Φ_m is the measurement objective function (4), μ is a weight factor, and Φ_r is the regularization objective function here defined as:

$$\Phi_r=\sum_{i=1}^{n_{kpar}}\left(log\left(C_p*w_i^a*T_{2i}^{*2}\right)-log\left(k_{mf,i}\right)\right)^2 \tag{6}$$

In (6), $k_{mf,i}$ is the estimate of the ith hydraulic conductivity value in $\boldsymbol{k}$, which is used by the groundwater model to simulate the hydraulic head responses of the aquifer test.

Minimization of Φ_t can be reformulated to the constrained minimization problem of simultaneously estimating the values of $\boldsymbol{p}_2$ and the weight factor μ that makes the contribution from the regularization term $\mu\Phi_r$ as small as possible and assures that Φ_m becomes less than or equal to a preset target value Φ_m^1. A reasonable value of Φ_m^1 can be judged from (4) by using knowledge or assumptions about the noise structure of the actual measurement data; the chosen value should be conservative (relatively high) to avoid overfitting. Using the Lagrangian multipliers technique, this constrained minimization problem can be reformulated to estimation of the values of $\boldsymbol{p}_2$, μ' that minimizes:

$$\Phi_t'=\Phi_r+\mu'\left(\Phi_m-\Phi_m^1\right) \tag{7}$$

Cooley and Vecchia [1987] gave an algorithm to solve this minimization problem. Figure 2 illustrates the flowchart of method 2.

The minimization problems of methods 1 and 2 are solved by using BeoPEST, a version of PEST [*Doherty*, 2010], which allows the inversion to be run in parallel using multiple cores and computers. We used a new version of BeoPEST modified and optimized by John Doherty particularly for our purpose to do gradient-based minimization involving several computational models (here a groundwater model and two geophysical models) with each their parameters, whereby different parts of the sensitivity matrix can be calculated by running just one or two of the models.

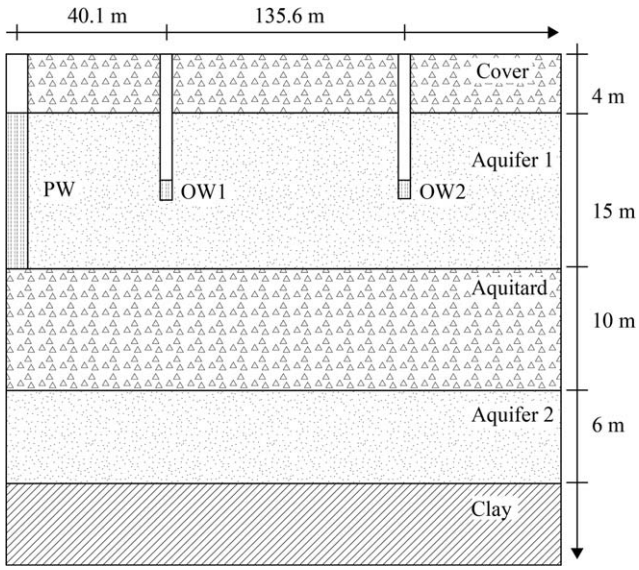

Figure 3. The geological model used to test the joint inversion methodologies (the parameter values are given in Table 1).

3. Synthetic Model Study

The test case comprises a confined two aquifer system where the aquifers are separated by an aquitard (Figure 3). The hydraulic and geophysical parameter values of the system are given in Table 1. For the petrophysical relation (3), the true parameter values are $C_p = 8.0 \times 10^{-3}$, $a = 1.0$. The test data set contains drawdown data from an aquifer test, full decay data from an MRS sounding, and the decay data from a TEM sounding (time derivatives of the vertical component of the magnetic field as a function of time). Finally, data weights applied in the inversion have been determined based on the true error model (described in the following).

During the aquifer test, groundwater was pumped from well PW for 21 days at a constant rate of 0.015 $m^3 s^{-1}$. The pumping well fully screens the upper aquifer. Drawdown of hydraulic head is monitored in two observation wells, OW1 and OW2 that partially screen the upper aquifer (Figure 3). The drawdown data were generated by adding random Gaussian noise with a standard deviation of 0.05 m to drawdown simulations made using the true hydraulic parameter values. The distribution in time for the data corresponds to one measurement at the end of each groundwater model time step, resulting in 70 measurements for each observation well.

The MRS sounding data were simulated using the true geophysical parameter values and a single turn 100 m by 100 m square loop (both as transmitter and receiver), 24 pulse moments with 20 time gates each, assuming a typical dead-time (instrumental delay between excitation termination and measurement start) of 40 ms. Data were contaminated with noise, according to the noise model suggested by *Auken et al.* [2008], which has two sources: data noise, amounting to a certain percentage of the true data value, and Gaussian noise. The MRS data set used in the test has 3% of data noise and the Gaussian noise is 10 nV before gating.

The TEM data set was simulated for a conventional square central 40 m by 40 m loop configuration. The data containing 36 time gates were contaminated by 2% of data noise. No Gaussian noise was added since the last datum was measured no later than the 1 ms boundary [*Auken et al.*, 2008].

Since none of the wells reach through the aquitard, we assume that its thickness as well as the presence of the deep aquifer 2 are unknown and can only be revealed by the geophysical data and the late part of the drawdown data monitored in the upper aquifer. Contrary to this, we assume that the boreholes inform about layer thicknesses and rock types of the cover and the upper aquifer 1, and that this can be used for the inversion to set initial values and limits for the geophysical parameters of these layers that are relatively close to the true values (Table 2). No limits were used for parameter values of the deeper layers, and their

Table 1. True Geophysical and Hydraulic Parameter Values for the Five Geologic Units Comprising the Synthetic Test Model

	Cover	Aquifer 1	Aquitard	Aquifer 2	Clay
Resistivity (Ωm)	30	250	30	250	5
w (-)	0.30	0.20	0.30	0.20	0.30
T_2^* (s)	0.10	0.25	0.04	0.60	0.02
Hydraulic conductivity (m/s)	2.4e-5	1.0e-4	3.84e-6	5.76e-4	9.6e-7
Specific storage (1/m)	5.0e-6	5.0e-6	5.0e-6	5.0e-6	5.0e-6
Layer thickness (m)	4	15	10	6	–

Table 2. Initial Values of Geophysical Parameters and Limits Imposed on Them During the Inversion[a]

	Cover	Aquifer 1	Aquitard	Aquifer 2	Clay
ρ					
Lower limit	20	100	10	–	–
Initial value	50	150	50	100	20
Upper limit	60	400	100	–	–
w					
Lower limit	0.01	0.15	0.01	–	–
Initial value	0.07	0.15	0.07	0.1	0.1
Upper limit	0.5	0.35	0.5	–	–
T_2^*					
Lower limit	0.05	0.15	–	–	–
Initial value	0.07	0.30	0.07	0.1	0.1
Upper limit	0.2	0.55	–	–	–
Thickness					
Lower limit	–	–	1	1	–
Initial value	Fixed	Fixed	7	7	Fixed in groundwater model
Upper limit	–	–	–	–	–

[a](–) indicates that no limits were imposed on the parameter value.

initial values were set relatively far from the true values (Table 2). The initial parameter values for hydraulic conductivity were chosen so that the initial value of the regularization term (6) was naught.

3.1. Results

The inversion results for both joint and individual inversions are presented in Figure 4 and Table 3. The figure is subdivided into four plots containing estimates of water content $[w]$, hydraulic conductivity $[k]$, decay time $[T_2^*]$, and resistivity $[\rho]$, respectively. Each plot shows the true values (black diamonds), values

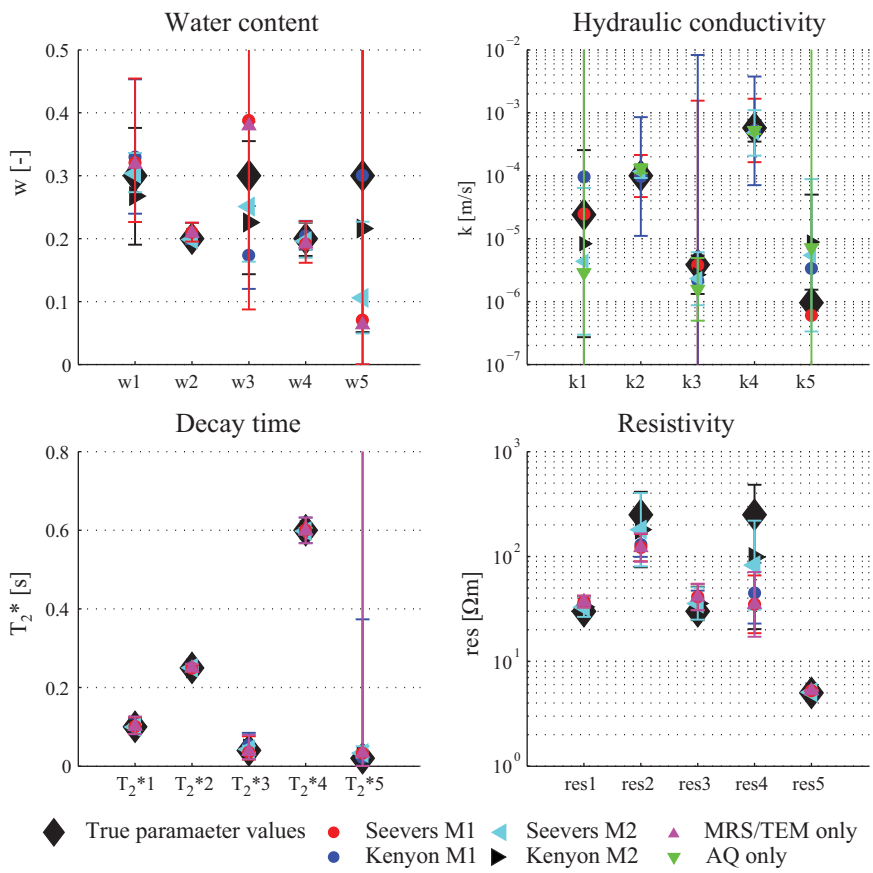

Figure 4. Inversion results using the synthetic data set.

Table 3. Estimated Layer Thicknesses: The Estimated Value is Shown First, While Its Standard Deviation is Given in Brackets

	True	MTS/TEM	Seevers M1	Seevers M2	Kenyon M1	Kenyon M2
Aquitard	10 m	9.50 m (0.37)	9.51 (0.35)	9.90 (0.25)	9.76 (0.25)	9.95 (0.27)
Aquifer 2	6 m	6.75 m (0.62)	6.75 (0.58)	6.17 (0.48)	6.30 (0.41)	6.10 (0.47)

estimated by method 1 (circles), values estimated by method 2 (left and right triangle), values estimated only on basis of the MRS and TEM data sets (upward triangle), and only aquifer test (downward triangle). The results from the independent aquifer test inversions were obtained using the true layer thicknesses for all layers. Having fixed the layer thicknesses in the independent aquifer test inversion is in the present case equivalent to estimating the transmissivity of the layer. However, to simplify the explanations in the following, we will be naming the estimated parameters "hydraulic conductivity." We have chosen only to show one set of hydraulic conductivities (k_{mf}) for method 2 since regularization had reduced the difference between values derived from MRS and those obtained from aquifer test data to almost zero. Confidence intervals of parameter estimates are shown for the independent inversions as well as for method 1. The confidence intervals for the hydraulic conductivities obtained using method 1 was calculated by propagating parameter uncertainties from the hydrological parameters through the petrophysical relation. For method 2, standard statistical methods do not apply due to the formulation of the regularized inversion. To approximate the parameter uncertainties, we therefore ran multiple inversions over the data set using different noise perturbations. Due to the computational burden of this methodology, we were however limited to 100 realizations. The estimated confidence intervals should therefore only be thought of as guiding.

The results presented in Table 3 show that layer boundaries are generally well estimated. The largest estimation error is for the thickness of aquifer 2, which is overestimated. The depth to the bottom of aquifer 2 (which is the top of the clay layer) is estimated very accurately because the TEM data are very sensitive to the low-resistivity clay layer (this can be acknowledged by adding the two estimated layer thicknesses for each inversion). The resistivity of the clay layer (res5) is also estimated accurately for the same reason whereas the resistivity estimates are less accurate for the high-resistivity layers (the two aquifers—res 2 and res 4; see Figure 4). This is as expected. The resistivities estimated only on the basis of the geophysical data set ("MRS-TEM" curve in Figure 4) are similar to estimates obtained by both joint inversion methods no matter which petrophysical relation applied. Therefore, in this case, joint inversion does not improve the accuracy of estimated resistivities and layer depths.

Regardless of the inversion method or the petrophysical relation applied, the estimated water content is in accordance with the true values for the aquifers (w2 and w4) and decay times are in accordance with their true values for both aquifers and aquitards (Figure 4).

The C_p estimates are 7.57×10^{-3} and 5.86×10^{-3} using Seevers relation and methods 1 and 2, respectively, which are close to the true value of 8.00×10^{-3}. Using Kenyons relation the C_p estimates are 0.83 and 1.70 for method 1 and 2, respectively.

The estimates of hydraulic conductivity are also close to the true values when using Seevers relation and either of the joint inversion methods. The estimated uncertainties are, however, larger for the joint inversion methods. This originates from the fact that the uncertainties from the individual terms in (3), namely the water content, decay time, and C_p have to be propagated through the linearized version of the petrophysical relation. Doing this, the combined uncertainties for all these terms tend to be higher than that of the hydraulic conductivity estimated based on pure aquifer test analysis.

It is apparent from Figure 4 that through detailed information of layer thicknesses (either from well logs or geophysical inversion), accurate estimates of hydraulic conductivity can be obtained for the aquitard and aquifer 2. This originates from the leakage through the aquitard and the flow in aquifer 2 caused by the long duration of the aquifer test. To minimize this assistance in the aquifer test inversion, the duration of the aquifer test used in the analysis was reduced so that the drawdown data set was dominated by the hydraulic properties in the upper aquifer the results from this inversion setup can be seen in Figure 5.

Doing this, we find that independent aquifer test inversion can only accurately estimate the hydraulic conductivity of the upper aquifer (k2), while the estimated leakage factor of the aquitard (k3) and the hydraulic conductivity of the lower aquifer (k4) are erroneous and undetermined. However, when applying either of

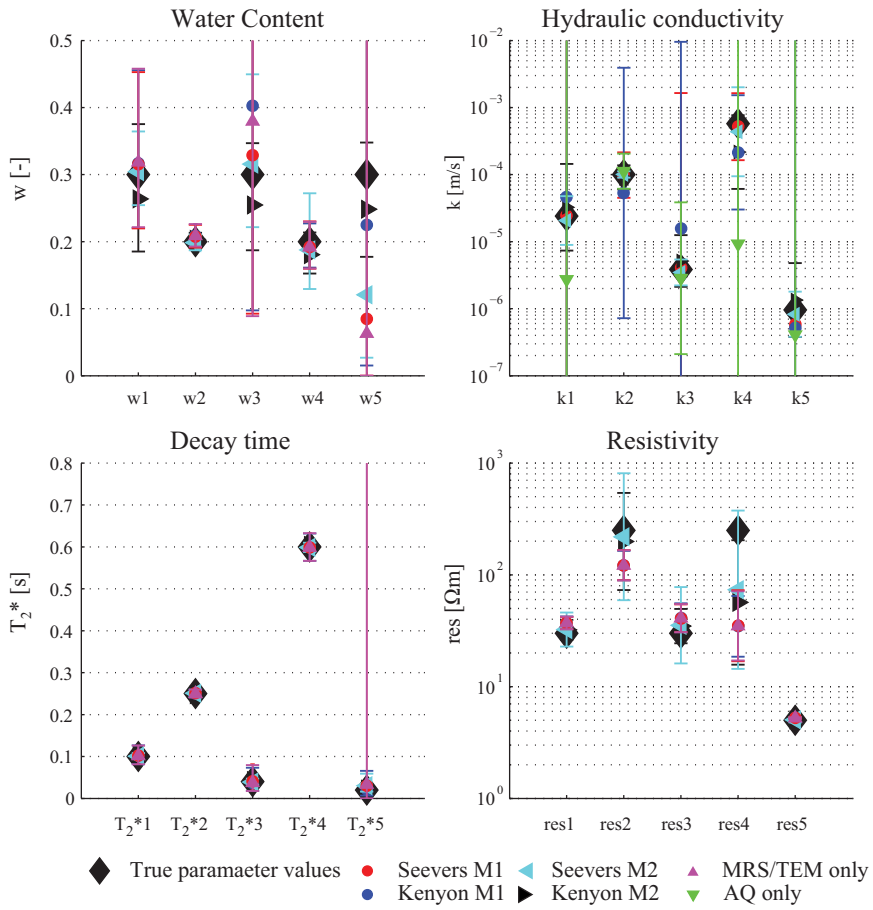

Figure 5. Estimated parameters based on the aquifer test data set of reduced duration.

the joint inversion methods, these parameters are estimated more accurately and with less uncertainty. Again, using Seevers relation gives the most accurate results, but also Kenyons relation give more accurate results than the individual aquifer test inversion.

4. Field Application

The field data used for demonstration of the joint inversion methodologies originate from the Platte River Valley, Nebraska [*Irons et al.*, 2012; *Payne and Teeple*, 2011]. The data set comprises TEM, MRS, and aquifer test measurements collected at a location named site 58 northwest of the town of Lexington [*Irons et al.*, 2012]. All available boreholes at this site can be seen from Figure 6, while Figure 7 outlines the geology and shows the screen intervals of the wells that were selected for the aquifer test analysis.

4.1. Hydrogeological Setting

The aquifer system at this site is a part of the High Plains Aquifer system covering parts of Wyoming, South Dakota, Nebraska, Colorado, Kansas, New Mexico, Oklahoma, and Texas [*Gutentag et al.*, 1984]. At site 58, this aquifer system mainly comprises two aquifers, one found in the deeper Tertiary Ogallala group, and one found in the shallower alluvial deposits, respectively. In the following, these groups will be referred to as the Ogallala and the Alluvium, respectively.

The Ogallala mainly consists of a poorly sorted mixture of sand, silt, clay, and gravel [*Gutentag et al.*, 1984]. It is mainly unconsolidated or weakly consolidated, but contains sandstones at some depths. The thickness of the formation varies between 30 and 150 m, with a thickness of approximately 88 m at site 58 (Figure 7). The hydraulic conductivity is expected to lie within the interval from 5.6×10^{-7} to 5.9×10^{-5} m/s [*Irons et al.*, 2012].

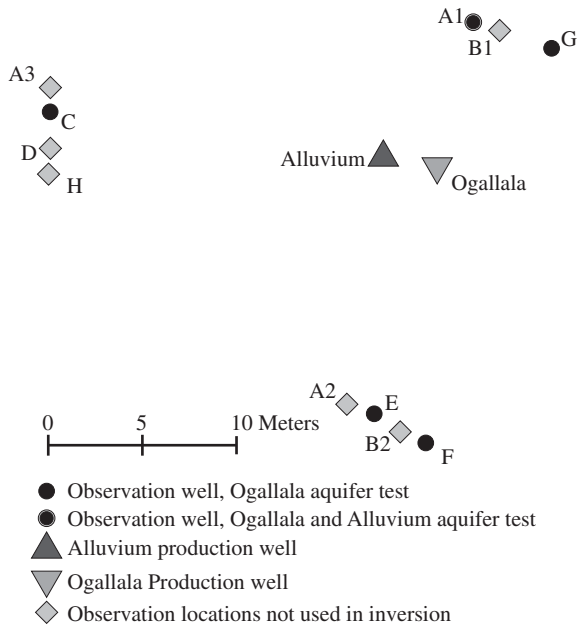

Figure 6. Location of pumping wells and observation wells at site 58 [after *Irons et al.*, 2012].

Quaternary deposits of silt and clay directly overlay the Ogallala at most locations and separate it from the more shallow Quaternary Alluvium formation. These silt and clay layers form a leaky confining unit between the two aquifers. Its thickness varies between 3 and 35 m within the regional area, and locally around site 58 it is approximately 14 m (Figure 7).

The Alluvium comprises sand and gravel units with occasional interlayered silt and clay layers. It acts as an unconfined aquifer with a high hydraulic conductivity between 1.1×10^{-3} and 1.6×10^{-3} m/s, which together with its shallow depth makes the Alluvium the most exploited aquifer in the area.

4.2. Data and Observations

The MRS data used in this study were collected by the USGS in 2010 using the Vista Clara Inc. GMR instrument with a 100 m square loop [*Irons et al.*, 2012]. The data were processed using the method of *Walsh* [2008]. Data points used in the inversion were selected using logarithmic time gates; compared to linear time gates, this give the most reliable results [*Nielsen*, 2012].

TEM data were collected by the USGS in 2007/2008 using both the Protem 47 and Protem 57 equipment [*Payne and Teeple*, 2011]. The data used in this study were collected using a 40 m square loop with three different moments for both the Protem 47 and the Protem 57 transmitters. For a full description of the TEM data set, we refer to *Payne and Teeple* [2011]. We processed the data using the SiTEM software [*Nielsen*, 2012].

Two aquifer tests were conducted in February to March 2010 [*Irons et al.*, 2012]. Both tests lasted for 192 h, with 96 h of pumping and 96 h of recovery. Only the drawdown data are used in the present analysis. During one test, groundwater was pumped from a well screening the Alluvium (Figures 6 and 7). The pumping rate was measured to change nine times during the pumping period, varying between 0.0308 and 0.0386 m^3/s. Drawdown was measured in observation wells A1, A2, and A3 (Figure 6) that all screen the Alluvium. Comparison of the three drawdown time series showed that the alluvium is not quite behaving as a homogeneous aquifer. Since the groundwater system is modeled here as consisting of laterally homogeneous layers, we chose to use the drawdown data measured in well A1 only. Furthermore, the first 30 s of drawdown data had to be excluded from the analysis because they were obviously affected by wellbore storage, which is not simulated by the groundwater model.

During the other test, groundwater was pumped from a well with a 100 m screen covering the Ogallala (Figures 6 and 7). The pumping rate was not recorded at the beginning of the test, which is unfortunate as the drawdown data clearly show that the rate significantly decreased between 2 and 5 min of pumping. We therefore had to exclude the first 5 min of drawdown data from the analysis. From then on the rate stabilized with only small variations between 0.0057 and 0.0059 m^3/s. Drawdown was measured in all observation wells (Figure 6), but in the following analysis, we only used data from the five wells shown in Figure 7. (Data from wells B1 and B2 were excluded because these wells have long screens, which are difficult to "observe" in a numerical model; data from wells D and H were excluded because they are similar to the data from well C, which screen the same layer as wells D and H; and data from wells A2 and A3 were excluded for the same reason mentioned for the first pumping test.)

4.3. Modeling and Inversion Setups

The aquifer test responses were simulated as radial flow using the MFCG code [*Clemo*, 2002] described previously. The model discretizes the 10 lithological layers shown in Figure 7 into 23 numerical layers.

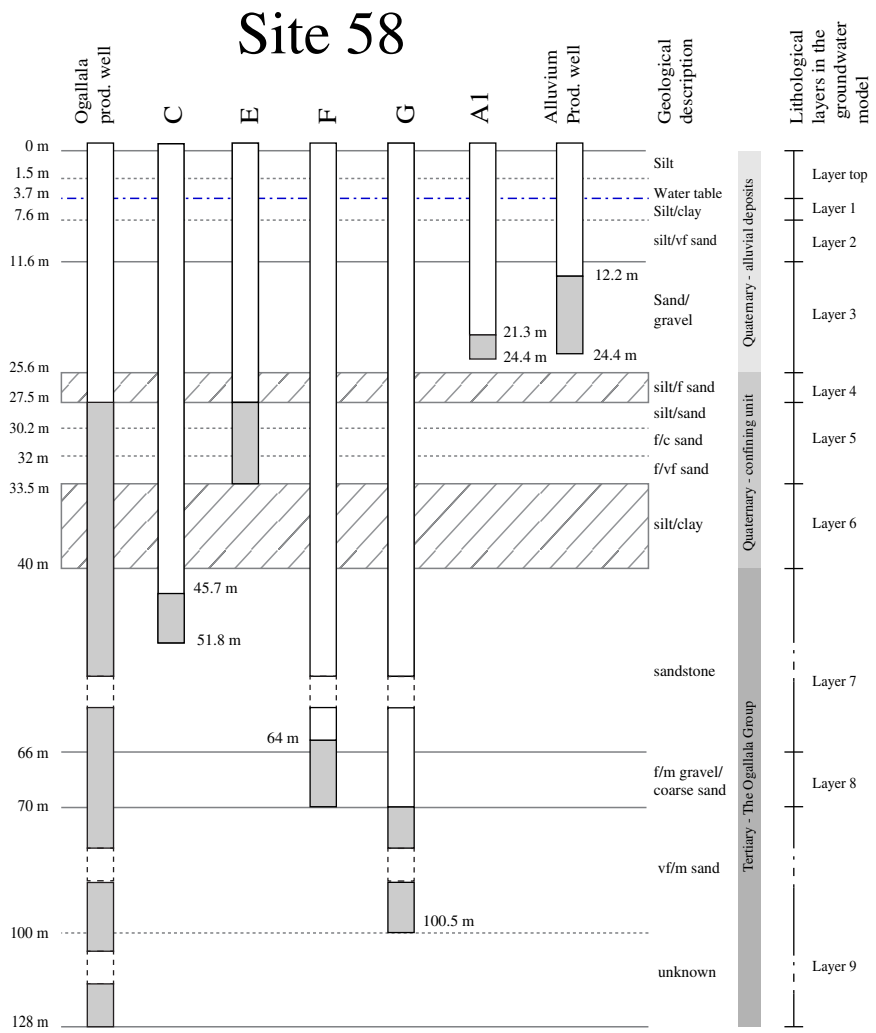

Figure 7. Geology at site 58 and screen levels for pumping and observation wells used during the aquifer tests (modified from *Nielsen* [2012]). Not in scale.

Numerical layers that subdivide a specific lithological layer are defined to share hydraulic properties. Flow is not simulated in the unsaturated zone ("Layer top" in Figure 7), while it is simulated as confined in all other layers. The specific storage of "Layer 1" was fixed at a value that, multiplied by the layer thickness, gives a storage coefficient of 0.2 for the layer in order to approximately simulate the influence of the water table. This approximation was judged to be reasonable since a sensitivity analysis showed that the modeled responses were only marginally affected by this parameter. The layer thicknesses are also fixed during the inversions since they can be determined from the cutting logs from the boreholes [*Irons et al.*, 2012]. For each of the nine deepest layers, both the hydraulic conductivity and the specific storage are defined to be parameters to be estimated by inversion.

Similar to the groundwater model the MRS and the TEM models have a fixed layer structure. Again the layer boundaries were chosen based on the cutting logs, such that they equal the hydraulic units defined in the groundwater model. The water content and decay time of the nine deepest layers in the MRS model were defined as parameters, and similar resistivity parameter was assigned to each of these layers.

Four inversion setups have been tested: (i) an independent aquifer test inversion where data from the two pumping tests are inverted simultaneously; (ii) joint inversion of MRS and TEM data; (iii) joint inversion of all data sets using method 1; and (iv) joint inversion of all data sets using method 2.

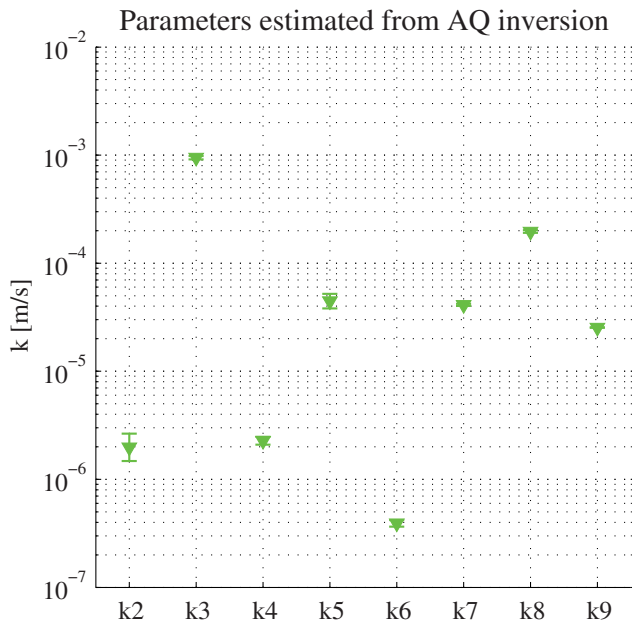

Figure 8. Parameter values determined based on inversion of both aquifer test data sets.

The independent aquifer test inversion was possible in this case because drawdown data are available at most depths and because layer parameterization can be based on the lithological information that is available from the surface to the bottom of the system.

The joint inversion of MRS and TEM is limited by the penetration depth of the methods. At site 58, the penetration depth could be determined based on sensitivity analysis to be approximately 60 m for the MRS method using a square 100 m loop [*Irons et al.*, 2012]. This corresponds approximately to the top of layer 7 in the model (Figure 7).

The data weights applied in the inversion are determined using a standard deviation of 0.05 m for the aquifer data, for the TEM data set, the weights were determined from processing, and for the MRS data set, the method applied was that of *Auken et al.* [2008] with a uniform Gaussian noise of 3% of the data value.

The joint inversion setups were tested using two versions of the petrophysical relation (3): either $a = 4$ "Kenyons relation," or $a = 1$ "Seevers relation."

All inversions were initiated using the same initial parameter values except for the C_p value, which was adjusted manually to secure good initial accordance between the MRS and the groundwater model parameters (making the initial regularization term value practically nil).

4.4. Results

The analysis was initiated with a sensitivity analysis to determine which parameters could be included in the inversion. Using the two aquifer test data sets, it was found that hydraulic conductivities could be estimated for layers 2–9, and three storage parameters could be determined namely for the upper aquifer (layer 3), one for the sand layer in the aquitard (layer 5), and one for the Ogallala aquifer (layers 7–9). The estimated hydraulic conductivities from the independent aquifer test inversion can be seen from Figure 8. The estimated storage parameters are not shown since they are unimportant to the following discussion.

Based on the narrow confidence intervals, it is evident that using the aquifer test data set results in estimates of hydraulic conductivity with low uncertainty for the eight deepest layers of the system. The estimated values for the Alluvium are in agreement with previous studies at the site [*Irons et al.*, 2012] with hydraulic conductivities of 9.6×10^{-4} m/s. The estimated values for the Ogallala range between 2.5×10^{-5} and 2.0×10^{-4} m/s. The upper level for this range exceeds that reported by *Irons et al.* [2012], but, the high value is estimated for a layer which according to the well log comprises gravel or coarse sand, and it therefore appears to be reasonable.

Figure 9 shows the estimated parameters from the independent MRS/TEM inversion. Based on a sensitivity analysis, we found that only a subset of the total geophysical parameter vector can be estimated; namely, the parameters pertaining to sand, gravel, and sandstone layers. The parameters pertaining to layers mainly comprising silt or clay (layers 4 and 6) could not be estimated during the inversion due to low sensitivity. The sensitivity analysis also shows that decay time and water content cannot be determined for layers 8 and 9. Instead of fixing these parameters in the inversion it was decided to tie these to the corresponding parameters pertaining to layer 6. Thereby, only one set of parameters was estimated for the Ogallala.

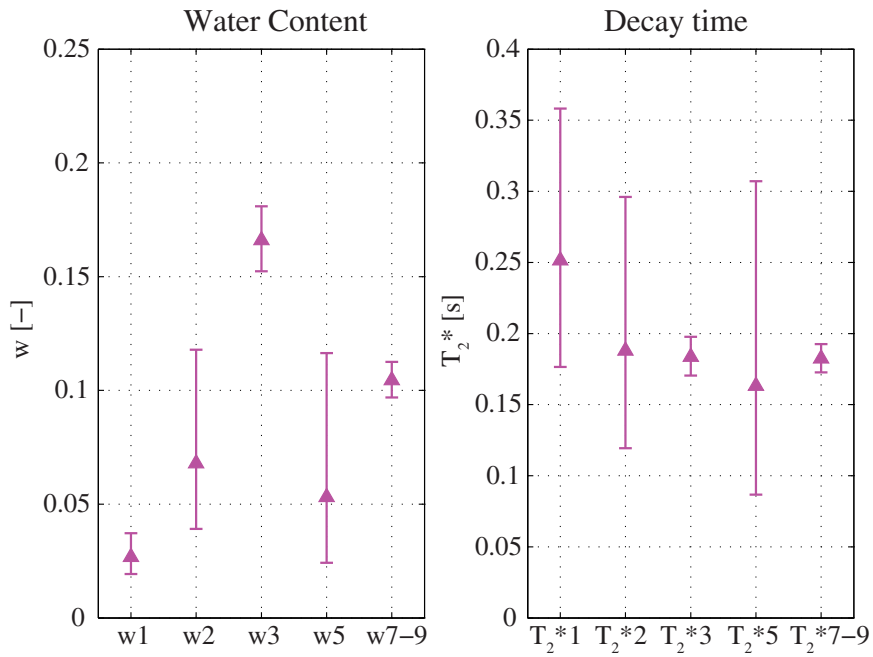

Figure 9. Parameters estimated from independent MRS/TEM inversion. Fixed parameters (layers 4 and 6) are not shown on the plot.

The linear confidence intervals in Figure 9 show that the smallest uncertainty is associated with the estimated parameters of the two aquifers, whereas estimates pertaining to more fine-grained materials are more uncertain. This is as expected. Since the data show relatively high sensitivity to both the hydraulic and geophysical parameters of the aquifer layers (layers 3 and 7–9), it is expected that these will dominate in the joint inversion, hereby effectively determining the C_p parameter in the petrophysical relation.

Based on the sensitivity analysis and individual inversions results, we defined the parameterization of the joint inversion setups as follows. Through the petrophysical link between the hydrological model and the geophysical model, the sensitivity to the geophysical data set can assist the estimation of otherwise insensitive hydraulic conductivity parameters. During the sensitivity analysis of the individual aquifer test, we found that the hydraulic conductivity of layer 1 could not be determined. However, since the MRS data set showed sensitivity to the water content and decay time of this layer, a hydraulic conductivity for this layer can be determined through the petrophysical link between the models.

For layers to which the geophysical data are insensitive to both water content and decay time, aquifer test data will do little to assist the determination of these due to the fact that both water content and decay time are variables of the petrophysical relation. For these layers, assumptions must be made about the geophysical parameters, or they have to be tied to more sensitive parameters expected to have similar values. By the independent geophysical inversion, we found that MRS parameters could not be estimated for layers 4, 6, 8, and 9. For layers 4 and 6, this is most likely due to the fast decay of the MRS signal in the fine-grained material, and for layers 8 and 9 it is due to the penetration depth. Therefore, for layers 4 and 6, the water content was assumed to be 0.3 (as an acceptable value for such geological units), while the decay time was estimated as a part of the inversion. For layers 8 and 9, the water content was tied to that estimated for layer 7, while decay times were estimated for each of these layers.

Figure 10a shows the estimated water contents using the four different joint inversion setups. The water contents estimated using method 1 are more variable and extreme than the estimates obtained by the joint inversion of MRS and TEM only (Figure 9a, layers 3 and 5). Compared to method 2, the parameters estimated by method 1 also appear to be more sensitive to the choice of petrophysical relation. The fixed petrophysical link using method 1 also appear to affect the fit to the MRS data set (Table 4), which is worse than the individual inversion or the inversions using method 2.

The same can be said for the estimated decay times shown in Figure 10b. The estimated values for layers 8 and 9 are caused by the forcing of the groundwater model and the aquifer test data through the

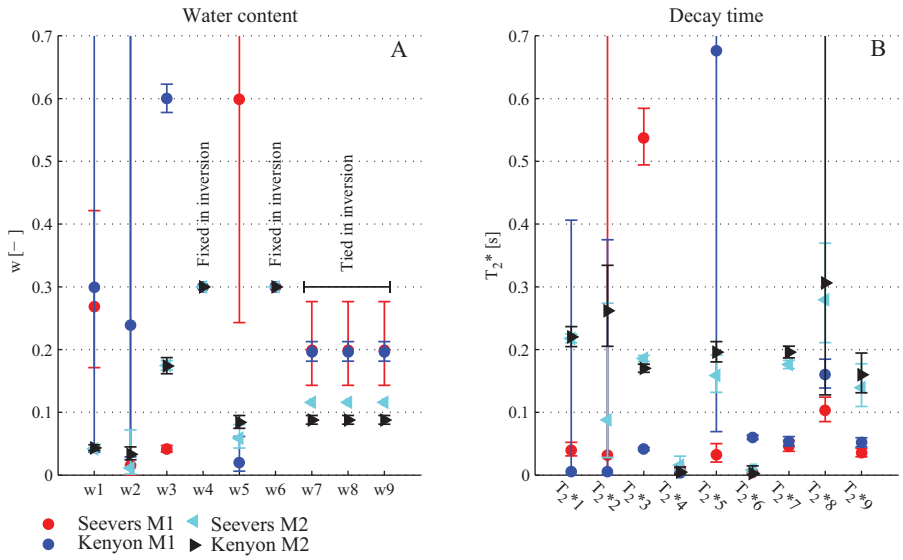

Figure 10. Geophysical parameter estimates from the joint inversion using both formulations of the petrophysical relation and both methods 1 and 2.

petrophysical relation using water content estimated for layer 7, since the MRS data have limited sensitivity to the parameters at these depths. An important difference in decay time estimates is observed for the upper part of the Ogallala aquifer (layer 7). Using method 1 gives a decay time estimate of <0.1 s, which corresponds poorly to the type of deposits (sand/sandstone) found in the well log. Here the estimate of ~0.2 s obtained using method 2 appears much more realistic.

Figure 11 shows hydraulic conductivities estimated using the four joint inversion setups. Confidence intervals for k estimates using method 1 are calculated by propagating parameter uncertainties through the petrophysical relation, and for method 2 they have been estimated by the same methodology as for the synthetic model. However, the noise perturbations have been added to a data set created using the optimized parameters from the inversion.

The estimates are most variable for method 1. Using method 1 and Kenyons relation, results in hydraulic conductivity estimates, which generally deviate from the other joint and independent inversion estimates. There also appear to be a correlation between hydraulic conductivity estimates of layers 5 and 6 (hydraulic conductivities of these two layers had a correlation coefficient of −0.99 in the independent aquifer test inversion).

Only the k_{mf}-estimates (hydraulic conductivities used by the groundwater model to fit the aquifer test data) are shown for method 2, because the k_{mrs}-estimates (computed from the MRS parameter using either Kenyon's or Seevers relation with the appurtenant C_p estimate) deviated only slightly from these and because the two corresponding estimates were within each other's confidence intervals. The k estimates are practically unaffected by the choice of petrophysical relation, and the fits to the data are also unaffected (Table 4). The only exception is for layer 1. However, also for the pure aquifer test analysis it was found that this

Table 4. Contribution to the Objective Function (Weighted Sum of Squared Residuals Calculated From (4)), Regularization Term (Sum of Squared Log Difference in Hydraulic Conductivity), and Estimated C_p Values for Six Inversions

Data Type	(i) Aquifer Test	(ii) MRS/ TEM	(iii) Method 1, Seevers	(iii) Method 1, Kenyon	(iv) Method 2, Seevers	(iv) Method 2, Kenyon
TEM	–	0.012	0.25	0.84	0.011	0.027
MRS	–	1.20	4.77	4.60	1.3189	1.4488
Aquifer test	29.36	–	22.92	19.07	29.945	32.378
Regularization (not weighted with μ)	–	–	–	–	1.06	0.10
C_p	–	–	0.0825	5.07	0.0236	24.27

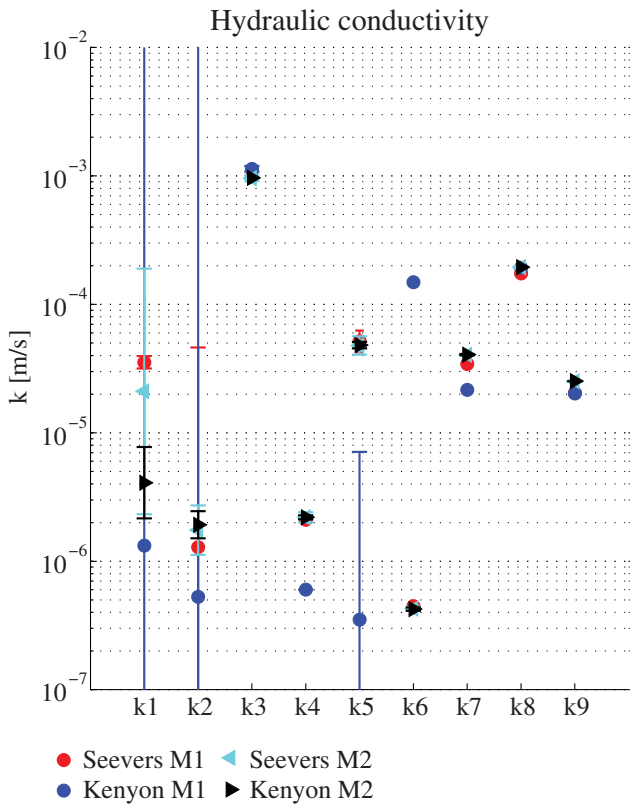

Figure 11. Hydraulic conductivity estimates from the joint inversion using both petrophysical relations and both methods 1 and 2. Confidence intervals are not determined for layers 4 and 6 since these *k* estimates are based on fixed water contents.

parameter has little influence on the simulated responses. The best accordance with the pure aquifer test-based estimates is seen for Kenyon's relation using method 2 (see Figure 11 and Table 4).

Table 4 shows the values of the three measurement objective function terms, the regularization term, and the C_p estimate obtained by the various inversions. Based on the formulation of the objective function, each of the individual terms of (4) should sum to 1 if the respective models closely represent the true hydrogeological and geophysical conditions at the site and that the used noise levels correspond to the actual measurement noise levels. The terms for TEM and aquifer test data are generally not near 1. For TEM, the values are <1, which either indicates overfitting of the data or that the noise level has been overestimated. For the aquifer test data, the values are >1, which either indicates structural error (that the used 2-D groundwater model with homogeneous layers cannot fully capture the true hydrogeological variation) or that the measurement noise assumed for the aquifer test data is too small. We consider the first explanation to be the most likely.

The first two data columns in Table 4 show the results from the individual inversion of aquifer test data and the joint inversion of TEM and MRS data, respectively. From these objective function values, we determined the target value $\Phi_m^1 = 30.5$ that was used for the method 2 joint inversions. Data columns three to six show the joint inversion results. For all joint inversions, the objective function values are comparable to those obtained by the individual aquifer test and geophysical inversions. For method 2 inversions, the small regularization term values show that good agreement can be obtained between hydraulic conductivities estimated to fit the aquifer test data and corresponding hydraulic conductivities computed from the estimated MRS parameters. However, based on the value of the regularized objective function the best accordance is achieved using Kenyon's relation. No matter the relation, method 2 produced fits to the data that are comparable to the fits of the individual inversions.

5. Discussion

5.1. Estimation of Parameters in the Petrophysical Relation

Both test cases demonstrate that either of the two joint inversion methods makes it possible to estimate the value of C_p together with the hydraulic and geophysical parameter values.

In the tests, we used a constant value for C_p for all layers of the system. This may be a simplification compared to real hydrogeological systems; some studies for example indicate that C_p may vary with the grain size [*Nielsen et al.*, 2011; *Plata and Rubio*, 2008]. Using just one C_p for a system where there rightly should be more will to some extent bias parameter estimation. In principle, the presented joint inversion methods allow estimation of individual C_p values for different parts of a hydrogeological system (e.g., clayey and

sandy units). Allowing C_p to vary between units will surely result in a lower value of the regularization term when using method 2 or improve the fit to the data when using method 1. However, it will also increase the uncertainty of the estimated C_p value(s), and reduce the constraint between the geophysical and the hydrological models. We therefore recommend to only using >1 C_p when this is either required to produce convincing model parameter estimates and data fits or firmly supported by other investigations.

In the present study, we found that a and C_p could not be determined independently. This is likely to be the case due to lack of independent data. Using the MRS methodology, the spectrum of differences in water content between geological units is small due to instrumental limitations. It is therefore unlikely that both a and C_p could be determined even in the case where multiple aquifer tests and multiple MRS soundings would have been available. This statement might change with future instrumental development.

5.2. Application of T_2^* as an Assumption of T_2

An apparent need for multiple C_p factors could also be explained by applying T_2^* as an approximation for T_2. The work of *Grunewald and Knight* [2011] shows that in the presence of magnetic field inhomogeneties, this assumption may be erroneous, especially for coarse-grained materials. This was also shown by *Knight et al.* [2012] for the same aquifer system in Nebraska as applied in this study. Here *Knight et al.* [2012] documented a complex link between T_2^* and T_2; where T_2^* was both faster (due to instrumental limitations and the inversion method) and slower (due to the physical properties of the free induction decay) than T_2 for the same borehole. Under such conditions using (3) to link the hydrological response to an MRS response would result in a biased parameter estimate using the proposed joint inversion methodology. However, we see this as a limitation of the MRS method and not a strong restriction of the proposed inversion methodology. The joint inversion methods presented will be as applicable with spin-echo and T_1 measurements where the need for assumptions about measured decay time in (3) would not be necessary. For the present study, we did not have T_1 data available so it is possible that what appears to be biased parameter estimates for joint inversion method 1 (Figure 10) could originate from a combination of a not optimal porosity exponent and a bias in the assumption of T_2^* being equal to T_2. The latter is most likely the dominant factor.

5.3. Estimation of Hydraulic Conductivity of a Hydrogeological System

Drawdown data from a pumping test are mainly sensitive to the transmissivity of aquifers and the leakance of aquitards (They are of course also sensitive to storativities, but this is irrelevant for the following discussion.) In the synthetic test case, the drawdown was observed in the upper aquifer for so long that it is possible to estimate the transmissivity of both the upper and the lower aquifers as well as the leakance of the aquitard by pure drawdown analysis. To estimate hydraulic conductivity from transmissivity or leakance of these units requires knowledge about their thickness, which in this case could be estimated on the basis of the geophysical data supplemented by information from the boreholes. It is therefore possible to estimate the hydraulic conductivity of the two aquifers and the aquitard without doing joint inversion; in this respect the geophysical inversion could have been done independently from, as well as jointly with, the aquifer test inversion. However, what the joint inversion did contribute in this case was the estimation of the C_p parameter value of the petrophysical relation on the basis of the full data set, which made it possible to also estimate the hydraulic conductivity of layers to which the aquifer test data are insensitive. In the synthetic test, we thus achieved quite accurate estimates of the hydraulic conductivity of the cover layer and the deep clay layer by the joint inversion. This does however only apply when a long duration aquifer test is analyzed. In the case where the duration of the aquifer test is reduced, the importance of the geophysical data increases.

For the field case, we used a fixed layer structure for the entire system. This was chosen since, compared to the synthetic case, we have boreholes reaching the bottom of the system (Figure 7) from which the lithology could be characterized [*Irons et al.*, 2012]. Moreover, the lower parts of the system are located at depths that were unreachable by the MRS measurements, and the system contains thin geological layers that can be important for the aquifer test responses but which cannot be resolved properly by the two applied geophysical methods. A comparison of Figures 8 and 11 shows that joint inversion by method 2 hardly changed the hydraulic conductivity estimates from those obtained by pure drawdown analysis. On the other hand, comparing Figures 9 and 10 shows that the estimated water content and decay time profiles did change somewhat by using method 2 instead of separate inversions, and this happened without deteriorating the fit to data (Table 3). The joint inversion thus not only helped by estimating the C_p parameter value, it also

caused new MRS parameter estimates that are in agreement with not only the geophysical data set, but also the aquifer test data. Hydraulic conductivity can likewise be changed by the joint inversions, as mentioned previously for the synthetic case. However, for our field case this only happened for layer 1 for which hydraulic conductivity could not be estimated from pure aquifer test analysis.

Another gain by the joint inversion is that the uncertainty of all types of data is allowed to play its role simultaneously and in balance during the inversion process. Furthermore, it makes it more straightforward to quantify the uncertainty of the estimated hydraulic conductivities than if the inversions are made separately.

5.4. Method 1 Versus Method 2

We have presented and tested two methodologies for doing joint inversion of MRS, TEM, and aquifer test data. Both methods worked similarly and well for the synthetic test case but differently for the field application. Their advantages and disadvantages in practical use will be discussed in the following.

Method 1 has the advantage of being simpler to set up. This is because it builds on the assumption of a perfect relation between hydraulic conductivity and MRS parameters so only the latter need to be estimated. The smaller number of parameters to be estimated eases the definition of the inversion problem and simplifies the input to be prepared for the inversion software (here BeoPEST). That the method is not regularization based also makes it straightforward to use standard regression-based techniques to quantify uncertainty of the estimated parameters, for example by computing confidence intervals. Uncertainties have also been estimated for the regularized inversion setup (method 2). However, this required perturbing the data set with different noise realizations. Such an approach is computationally very demanding compared to the standard regression-based methods. Method 1 is also limited by the assumption of a correct petrophysical relation, where correct means of the right type, but not perfect ("noise free"). In cases where the relation is biased method 1 is prone to produce what appear to be (possibly very) erroneous estimates. This is especially indicated by the results seen in Figure 10 for the field case.

Method 2 uses regularization between hydraulic conductivity and MRS parameters. This increases the number of parameters to be estimated, it gives a more complex formulation of the inversion problem, and a more complex input to the inversion software. Despite the higher number of parameters to be estimated, method 2 may run faster than method 1. This was the fact for both our demonstration cases. The explanation lies in the computation of the Jacobian (sensitivity) matrix, which is the costly part of all Jacobian-based inversion schemes. For method 2, this matrix computation can be divided between columns for which only the groundwater model needs to be run, and columns for which only the geophysical models need to be run. For method 1, all the models need to be run to compute all matrix columns. A disadvantage of method 2 is that its use of regularization makes it more computational demanding to quantify parameter uncertainty.

For the field case, we found method 2 to be less sensitive to the choice of petrophysical relation than method 1. Using Seevers' or Kenyon's relation with method 2 produced comparable estimates and data fits. The reason is that method 2 is regularization based, which only enforces hydraulic conductivity estimates to be in accordance with MRS parameter estimates to the extent that this is necessary to produce fits to data that are within the bounds of data uncertainty (quantified by the target value set for the measurement objective function term). This also means that method 2 will be more forgiving if parameters of the petrophysical relation are assumed to be constant, when they are actually (unknown to the hydrogeologist/geophysicist) variable with depth. It will also be forgiving if aquifer test data are to be jointly inverted with MRS measurements made at multiple locations among which the petrophysical relation parameters may vary spatially. This will not be the case for method 1. We therefore judge that method 2 will be the most useful method for the majority of practical applications.

6. Conclusion and Perspectives

This paper presented two methods for joint inversion of MRS, TEM, and aquifer test data. Both methods rely on the existence of a petrophysical relation between hydraulic conductivity and the MRS determined water content and decay time. For method 1, it is implicitly assumed that the functional form of the relation is perfectly correct ("noise free"), but that one or more parameters of the function need to be estimated together

with layer thicknesses, hydraulic conductivities, water contents, decay times, and electrical resistivities of the hydrogeological system. The implicit assumption simplifies formulation of the joint inverse problem, reduces the number of parameters to be estimated, and simplifies quantification of uncertainty of the estimates. However, computation of the sensitivity (Jacobian) matrix is more demanding for method 1, the method may cause bias in the parameter estimates when the assumed relation does not conform to the sediment type, and it is difficult to apply in an analysis containing aquifer test and MRS/TEM data sets collected at multiple locations.

Method 2 only uses the petrophysical relation as a regularization constraint. This has several advantages: it eases computation of the sensitivity matrix (which made method 2 run faster than method 1 for our synthetic test example as well as the field case), it causes less risk of large bias when the petrophysical relation is only approximately correct, and it makes the method forgiving for wrong assumptions about the form or spatial variation of the petrophysical relation. The method is thus also directly applicable for analysis of data collected at multiple locations in a system where the geophysical parameters are expected to vary laterally. This is especially important when this methodology is to be applied with a 3-D groundwater model.

Use of the methods was first demonstrated by a synthetic test case of a 40 m deep five-layer system with two aquifers and three semipermeable layers. The synthetic data were drawdown data monitored only in the pumped aquifer, and MRS and TEM sounding data reaching the deeper layers; all data were contaminated by noise of realistic magnitudes. Two scenarios were analyzed: one where the petrophysical relation was made to be perfect and other where it was biased. Each relation had three parameters, a scalar and two exponents, of which the scalar was considered unknown while the exponents were fixed. The scalar was therefore estimated together with thickness, hydraulic conductivity, water content, decay time, and resistivity of each of the five layers. Using the true (assumed known) petrophysical relation, the two joint inversion methods produced parameter estimates in accordance with the true values used to generate data, also for the deep layers. However, when applying the biased petrophysical relation, the estimates turned out slightly biased for the deeper part of the system without deteriorating the fit to data.

The methods were also applied to a set of field data collected by the U.S. Geological Survey in Nebraska, USA. The data were analyzed by assuming porosity exponents of the petrophysical relation similar to those suggested by *Seevers* [1966] or by *Kenyon et al*. [1988]; the scalar of each relation was estimated jointly with the other parameters. Using method 1 showed that the data fit deteriorated by assuming Seevers exponent, and mainly the geophysical parameter estimates turned out to be different from the other inversion results. Assuming Kenyon's relation produced good fit to data, and the parameter estimates were in better accordance with the other inversions as well as with the types of sediment found in the well log. When using method 2, the data could be fitted using both Seevers and Kenyon's relation. We judge that this forgiveness will make method 2 the more useful of the two joint inversion methods presented here. However, analyzing a data set by both methods may give a more complete insight into the information content of the data. For the Nebraska field data, we thus found that quite different profiles of MRS water content and decay time can fit the data equally well.

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