

Identification of Precious Artefacts: The Sonic Imprint for Small Artefacts

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Abstract: Identification of artworks is mainly based on a few characteristics which can be observed using non-invasive tools (sight, touch, simple instruments), the investigated properties being geometry, weight, colours, texture, etc. Nowadays, technology allows reproducing all these characteristics to such an extent that even expert conservators can be deceived: in particular at the present time even the geometry of an artwork can be easily reproduced with the help of laser scanner analysis and with a rapid prototyping machine or a computer numerical control (CNC) milling machine. We propose a new tool, the Sonic Imprint, producing a code capable of identifying a rigid artefact from its vibrational resonance frequencies beyond doubt. In fact the vibration modes of an artefact strongly depend on the spatial distributions of its density and elastic parameters, as well as on its internal defects, definable in terms of abrupt changes of elastic properties in a small portion of the object. Then even small differences of these properties (differences usually present even among “identical” objects produced with industrial methods, at least in terms of defects) give appreciable variations of the Sonic Imprint codes, allowing secure identification of artworks, prevention of clonation and even damage monitoring. Moreover the procedure is really robust, rapid, inexpensive and not invasive. We tested it on a large number of commercial objects with the same shape and dimension and on many artworks in archaeological museums: an example is described. The application of this methodology to small-size artefacts (from small stones, vessels, pottery to medium-large coins) involves some problems in the detection of the Sonic Imprint. The problems, just due to the smaller sizes of this kind of objects, arise from the presence of higher resonance frequencies and larger damping of the induced vibrations. This implies that probes and instrumentation should be replaced to be adapted to the new experimental conditions.

Keywords: Sonic imprint, resonance, Spectral analysis, Identification, Monitoring.

1. THE METHODOLOGY

The study of mechanical vibrations, and in particular the study of the frequency spectrum of the vibrations, has been extensively used during the last decades in a wide variety of scientific disciplines. Focusing our attention on the macroscopic point of view, the study of mechanical vibrations ranges from the earth scale, in seismology, to the millimetre scale in mechanical engineering. In particular, for instance, the spectral analysis of vibrations is used in microtremor study to characterize the dynamic response of subsurface soil to earthquakes [1, 2]; on a lower scale, in civil engineering, to predict damages on buildings due to seismic excitations [3, 4]; in mechanical engineering, for damage detection [5, 6]; in acoustics for modes prediction [7]. The Spectrum Analysis is also widely diffuse in physical chemistry in analyzing chemical oscillating systems [8].

The first idea of the Sonic Imprint methodology arose during the data processing of a survey carried out to retrieve a sonic tomography on an artefact (Niobidi crater, V century BC, [9]). It was noticed that the power spectra of the signals recorded by the sixteen piezoelectric sensors, applied on the surface of the artwork in order to measure the propagation times of elastic waves inside the medium, presented strong similarities in the resonance frequencies, despite of the different sensors positions and the different hitting points. The

fact that all the sensors measured, less or more, the same resonance frequencies is not surprising, because all sensors were attached to the same object, characterized by its elastic parameters distribution. But the surprising evidence was the strong repeatability of the resonant frequencies, changing sensor position and hitting point, also with a rubber hammer as vibration source. Further experiments on different objects with complex shapes confirmed this repeatability. Moreover, experiments on “identical” objects with simple shapes (like commercial dishes and vessels produced with industrial methods) evidenced the possibility to simple distinguish the objects from their resonance frequencies. The idea is then that the study of the resonances of an object is sufficient to distinguish it from other objects which apparently have the same shape, dimension and material. Furthermore, measurements of the resonant frequencies are practically not affected by variations of the positions of the vibrational sensors on the object and of the energizing points, neither by the amount of energy released by the excitation (in this sense, the methodology is very robust). The prediction of the vibrational frequencies of an object with a complex shape and composed by an inhomogeneous material is really a challenge: usually mode shape analysis is used only for uniform objects of simple shape [10] or for objects with a particular shape but non uniform [11, 12]. The idea is then to consider a work of art – such as a statue, a vessel, a plate, a piece of pottery, etc. – to be a black box endowed with special identifiable vibrational properties that can be detected reliably, quickly but also cheaply. Therefore, the application consists of checking

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some of these properties by measuring the free (or forced) vibrations of the artefact, regardless of type, amplitude or direction of the vibration modes involved. The methodology can be used both in frequency domain (forced vibrations) and in time domain (free damped vibrations), depending on the instrumentation available. In fact, for measurements in frequency domain we need a vibration source, like a loud-speaker system, driven by an amplifier with a quasi-linear response in the whole frequency range of interest (for large and medium artefacts, up to about 10000 Hz). As this is a severe constraint, the time domain acquisition is preferable, because it only requires a simple source, like a soft hammer or a pulse driven loudspeaker, and a subsequent frequency analysis.

The time domain methodology operation mode is as follows: a number of points (in our experience, eight are generally sufficient, representing a large portion of the artefact) needs to be selected on the surface of the artefact, after which some pictures must be taken so to remember all the locations. Furthermore, a set of points (for instance, five) has to be chosen where the artefact will be hit; taking into account that all the points will (more or less) trigger all the possible vibrations modes. However, strokes on points characterized by some symmetry of the artefact (for instance, the centre of a dish) tend to trigger vibrations modes having the same symmetry. Consequently, strokes on very asymmetrical points should generally be preferred, as they tend to trigger all the possible vibration modes.

Obviously the Sonic Imprint is not the only analytical methodology that allows the sure identification of an artwork by the comparison of analysis carried out at different times, but it presents some advantages over other methodologies. It looks at the artwork in a global way, and not only in a set of (more or less representative) points of the object: for instance colorimetry and spectroscopy often are able to characterize in a non-invasive way the surface of an artwork for the identification, but they work punctually, so that the repositioning of the probes is a critical point to compare the results of the test with those of the previous one. Furthermore the Sonic Imprint methodology, besides its simple and quick application, is safer than other methodologies which investigate artworks globally, like X-ray imaging. Finally the Sonic Imprint can be simply applied on a wide range of artworks regardless of the constituent materials and their homogeneity (the materials have only to be rigid) and is influenced by many physical parameters including size, density and relative dimensions of the artwork: on the contrary classical analyses (like colorimetry, spectroscopy, crystallography, mass spectrometry) are only able to recognize the constituent materials, so that only the presence of significant variability of composition markers assures the identification of the artwork (for instance the presence of a vein in a lapideous statue).

2. MATERIALS AND METHODS

A number of very light piezoelectric sensors should be placed on the artefact. For instance, the knocking sensor elements Murata Piezotite 6CC-10-3R9-1000, are non-resonant type sensors; they are very small and light resulting to be very useful for this application. They are sensitive to

radial accelerations and this should be kept in mind for all the subsequent tests. All the sensors have been connected to a multichannel Data Logger, namely the Mark 6 seismometer, ABEM Instruments AB, Sweden.

The sensor can be attached using very light elastic bands or very light plastic rings. The artefact should be de-coupled from its basement, unless the couple artefact-basement is also to be analyzed. If possible, the artefact should then be placed on a decoupling polystyrene or foam-rubber sheet or suspended in nylon net.

The artefact can be hit with a soft hammer – like a neurological reflex hammer – and the resulting vibrations should be recorded for about 200 ms. This is generally an adequate time for the damped vibrations to switch off in a large quantity of materials, although the time necessary for lapideous artefacts is often actually less than 100 ms. The sampling rate adopted in our experiments ranged from 25 μ s to 100 μ s, depending on the maximum frequency of interest in the analysis. With a sampling rate of 25 μ s the spectra of the signals can be calculated below 20kHz, the Nyquist critical frequency (Nyquist-Shannon theorem, [13]); below 5kHz with a sampling rate of 100 μ s. Recording break time can profitably be given by an external trigger driven by a mechanical (or solid state) switch integral with the hammer, or simply including in the data logger the trigger function as the overtaking of a selected threshold by the signals measured.

However, there is a critical, even if minor, point of the procedure: before calculating the spectra we have to cut out the first part of the signal of all the acquired signals in order to be sure that the signals contain only movements constituting free damped vibrations of the artefact. The first part of a signal is more or less influenced by both, the distance from the hitting point and the pulse waveform of the different hits. In our experience, a time period of 1-2 ms is sufficient to remove these differences, but for some (small-size) artefacts this time can be further reduced.

Each hit should be repeated several times, typically ten, so that a set of at least ten records for each signal can be selected (i.e. from the i_{th} hitting point T_{xi} to the j_{th} receiver R_{xj}) and subsequently used to filter any random noise (practically using the well-established “stacking procedure”, [14]).

Afterwards, we repeat the same procedure for all the hitting points, so that a set of different signals are collected. For instance, with 12 sensors and 5 hitting points, we get 60 different signals: actually, the 60 signals are repeated at least 10 times each, so that the complete set of signals is 600 or more.

In this study we then reduce the number of signals by using the stacking procedure (i.e., so as to average the various signals recorded, in particular those obtained with the same detector from the same hitting point) over the repeated signals (which, in principle, should be identical except for the random noise), thereby obtaining the previously mentioned 60 stacked signals. This operation preserves the signals (in fact they are in phase with each other) knocking out the associated random noises, which are obviously random and, consequently, self-extinguishing. Finally, we can calculate the corrected power spectra by means of an FFT algorithm [15]. Furthermore, these spectra have to be normalized (equal area for each spectrum). The spectra so obtained for

an artefact are often very similar to each other, especially if the artefact is characterized by a simple shape and regular symmetry. This has been recognized in the sets of data acquired on a variety of pieces of pottery (vessels, dishes, etc.).

In principle, the entire set of the normalized power spectra, together with the map of the hitting and receiving points (obtained by means of pictures), are the best signature we can get of the artefact, which can then be compared with future ones. In this case, however, we get a good amount of information but scattered over many similar spectra. In order to overcome this complexity, we can try to reduce the signature to one single spectrum.

In fact, even if the artefact is irregular, with a significant asymmetry (for instance, a statue), the obtained spectra are quite similar. This point was well observed in some very unsymmetrical statues although the comparison of the normalized spectra is generally surprising, due to their basic similarities, in particular if we take into account the main resonance frequencies involved.

Only a few times did we find two different groups of spectra: generally speaking when the geometry of the artefact was particularly intricate and the sensors placed, in such a way, as to pick up the different components of the acceleration or when we hit a point which excites particular vibration modes (typically, a point of symmetry). An additional cause of this problem arose when the material constituting the artefact was characterized by particular anisotropy (for instance, wooden artefacts).

In all these cases, however, we can follow the following procedures:

1. Divide the spectra in two (or sometimes three) families, and apply the following steps to each family, or
2. Eliminate the spurious spectra maintaining only the one main spectra group.

In both cases we have to recognize and label the various pairs (hitting point-receiver), which have been grouped (1) or excluded (2), in the final document. Afterwards, all the power spectra can be normalized. This is done to average the amplitudes regarding the different distances between the hitting points and sensors.

Finally, we average all the spectra to obtain a final representative spectrum. In the exceptional cases, that the families are two (or three), we will obtain two (or three) representative spectra. The Sonic Imprint code is composed by all the frequencies of the peaks of resonance of the average spectrum, but present at least in the 60% of all the spectra calculated from the signals of each sensor, together with the standard deviations of the frequencies, calculated from the spectra. Two Sonic Imprint codes are considered equal when the component frequencies coincide within the limits of their standard deviations.

In our experience the extraction of the set of frequencies composing the Sonic Imprint code do not present difficulties, in terms of repeatability of the spectra varying the sensor and the hitting positions, except for the identification of sub-groups of spectra (when such a characteristic is present).

A further development of the automatic production of the Sonic Imprint code can be offered by Chemometry, in term of multiway data analysis. In fact Multivariate Analysis could be applied on the whole of the acquired spectra, also in terms of classification and discriminant analysis: PLS-Toolbox software [16] is a very useful tool for this purpose.

The Sonic Imprint can also be presented in an agreeable way, i.e. by its transposition onto an analogical bar code, where the amplitudes of the resonances are expressed by the thickness of the bars. Obviously, the bar code is not very precise but very easy to read, so it can be presented to the visitors of a museum, near the correlated artwork, so serving as deterrent for criminals, too. An alternative way for recording the Sonic Imprint is to use an RFID card.

The first prototype of instrument dedicated to the automatic control of Sonic Imprint is presented in Fig. (1). The objects tested are the statues (made in marble, metals, arenites, etc.) vessels, pottery, etc., reaching in at least one dimension a minimum length of the order of 0.2-0.3 m (depending also on the constituting material), so that the resonant frequencies were in the range 0-6 kHz, i.e. the range in which this instrument works.

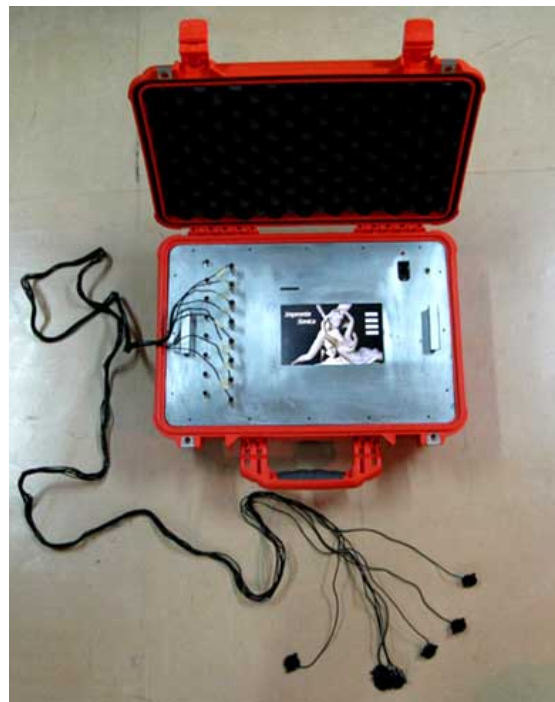


Fig. (1). The prototype of Sonic Imprint Tester (courtesy of Diasis s.r.l.). The instrument is equipped with sixteen probes.

3. RESULTS AND DISCUSSION

First experiments have been carried out both on commercial dishes, vessels and on precious artefacts, e.g. the Niobidi terracotta Crater (V century b.C.), Eleonora D'Aragona bust and Youth head (marble pieces, F. Laurana, XV century) and Saint Michele Arcangelo's painted marble statue (Gagini's apprentices, XVI century), antique artefacts (e.g. terracotta vessel representing Bes, an Egyptian god, IV century b.C.), two bronze busts (A. Ugo, 1930). Furthermore, two pieces of modern art have been analyzed, i.e. the statues Venus (pink

marble of Portugal, Sajeve 1994) and Origine (brass, Sajeve 1997), industrial plates and vessels, etc.

However, many experimental tests have been implemented [17, 18] in order to verify the general characteristics of the methodology as well as its possible limits. In particular, the test regarding the resolving power has been conducted using various practically identical artefacts. For instance two commercial glass vessels with same shape and dimensions (Fig. 2) exhibit really different resonance frequencies (Fig. 3). The robustness of the Sonic Imprint can be evaluated also from the repeatability of results by varying the position of the hitting point. In Fig. (4) it is presented the comparison of the analysis referred to the two hitting points on vessel A of Fig. (2): the resonance frequencies are stable within 5 Hz.

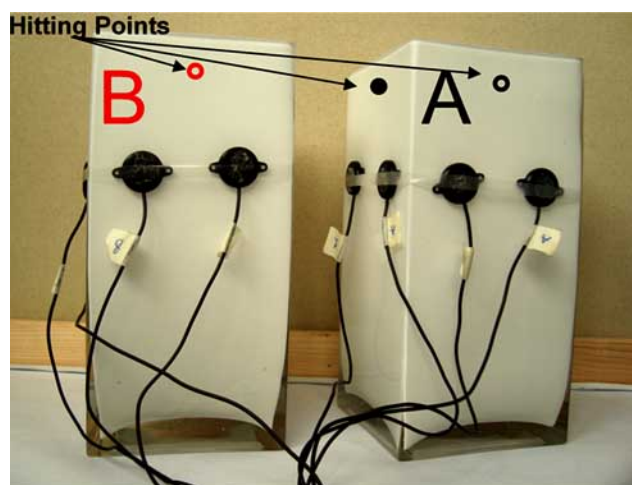


Fig. (2). Two commercial glass vessels with the same apparent shape and dimensions. The Sonic Imprints of the vessels were acquired with eight sensors. Circles indicate the hitting points (two different hitting points for vessel A and one for vessel B).

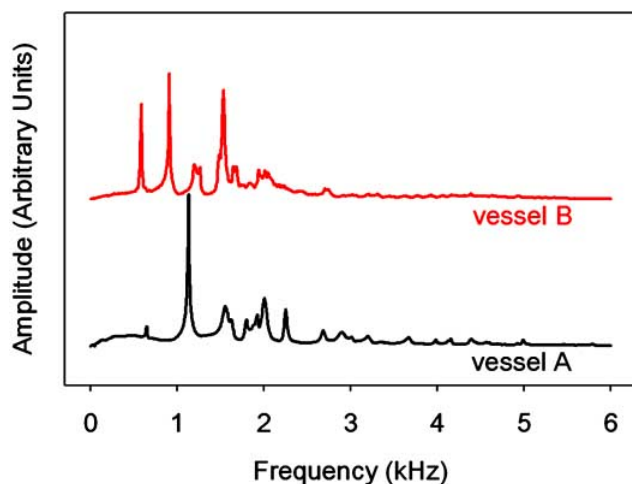


Fig. (3). Average spectra of damped vibrations from vessel A and vessel B of Fig. (2).

In spite of the identical appearance and substance, even twelve different dishes belonging to the same set of an Italian factory, exposed to different (central and external) hits of a rubber hammer and controlled by eight detectors placed

along the edge of the dishes, showed distinguishable Sonic Imprints.

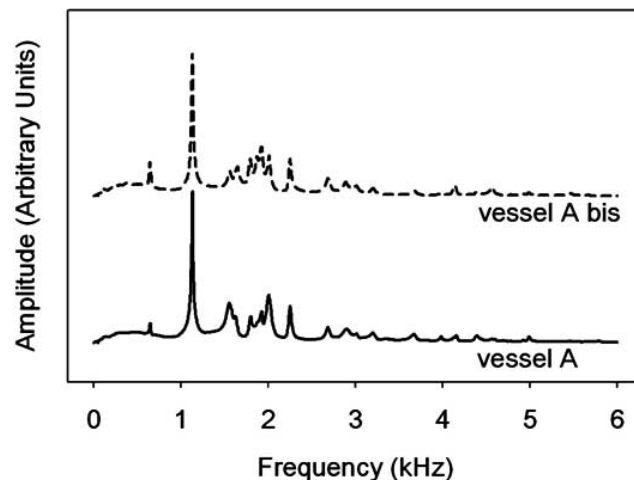


Fig. (4). Average spectra of damped vibrations from vessel A referred to the different hitting points indicated in Fig. (2).

Other tests have been carried out to control the influence of different energy sources (in particular two different hammers), two different hitting positions, one symmetrical – i.e. central – and the other asymmetrical – i.e. lateral. Another important test regarded the influence of different material to support the artefact during the acquisition of the Sonic Imprint. The tested supports were a foam-rubber sheet, an expanded-polystyrene support, a soft-wood tablet, a concrete cube and a suspended nylon net. The various Sonic Imprints retrieved using different energy sources and different supports presented the same frequencies (Fig. 5), but also small changes in the relative amplitudes of some frequencies. In fact, it is quite obvious that the central hits tend to trigger vibrational modes with a central symmetry, while the hits having no particular symmetry tend to trigger all the possible modes.

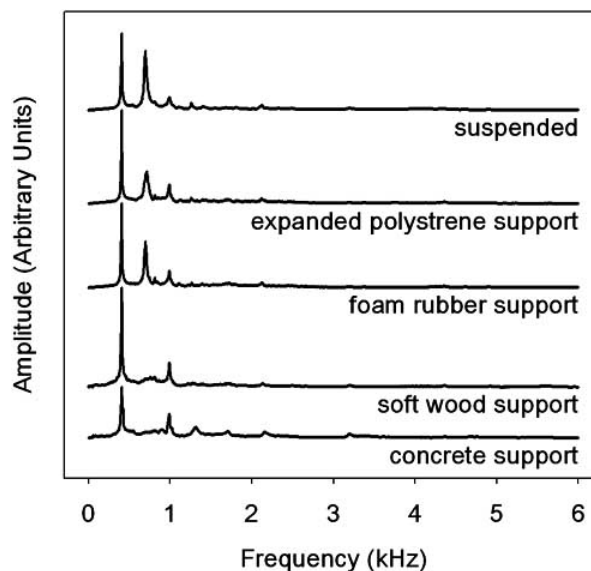


Fig. (5). The spectra presented in this figure were acquired using many different decoupling supports. Only relative amplitudes can slightly change (see ref. [17]).

Some other tests demonstrate the irrelevance of the position of the artefact during the test, if it is practically decoupled from its support. However, the information about the energy source used and the support conditions seems to be an important note to be saved in the archive file of the Sonic Imprint.

The Sonic Imprints of the twelve dishes have been also repeated using the same number of detectors and central symmetry, but changing the positions of the probes. These new Sonic Imprints, implemented to assess the stability of the methodology, showed the repeatability of the Sonic Imprint (similarity of the spectra of two successive tests on the same dish) and stressed important differences between the Sonic Imprint of the two dishes pertaining to the same industrial set. The results obtained with these tests stressed the high robustness of the methodology.

Another test has been carried out on a terracotta vessel, before and after causing a large and long crack. The differences of the imprints were evident and only a few resonance lines agreed.

This last test confirms that after large damages the Sonic Imprint is completely changed and the objects cannot be recognized by means of this procedure. The new imprint is richer in frequencies and with a trend to make the component frequencies higher (smaller sizes of the component pieces). Nevertheless, some frequency peaks (in the limits of possible errors) are still present, so that perhaps a careful analysis may be developed to recognize the eventual hidden identity.

Among the artworks recently checked we like to present in this paper the Sonic Imprint retrieved from the Landolina Venus (also called anadiomene), a wonderful statue of the 2nd Century AD, exhibited at the Archaeological Museum of Syracuse. This Sonic Imprint has been detected using sixteen sensors, distributed all over the body of the statue, and five hitting points which seemed to be representative, practically triggering most of the possible vibration modes.

Some phases of the measurements are presented in Fig. (6) where the light elastic strips, used to fasten the sensor to the surface of the statue, are clearly evident. In Fig. (7) the total spectrum obtained after the various stacking applications is presented. Finally the transposition of the Sonic

Imprint in terms of a bar code is presented in Fig. (8).

4. APPLICATION TO SMALL OBJECTS

In principle the dimensions of the object affect directly the main resonance frequencies of the free damped vibrations of the objects. To verify this assumption and to control its effect on the damped vibrations that give rise to the Sonic Imprint, we studied a sample of calcarenite parallelepiped having the dimension of about $0.34 \times 0.17 \times 0.072$ m (block A, Fig. 9). The measured velocity of longitudinal waves in such sample was 1550 ± 50 m/s that is a relatively low velocity. The weight of the whole block was about 5.74 kg. After the acquisition of the Sonic Imprint of the whole block A we cut it in two approximately similar blocks, respectively B (roughly $0.17 \times 0.17 \times 0.072$ m, 2920.5 g) and C (roughly $0.17 \times 0.17 \times 0.072$ m, 2811.4 g). Then we measured the Sonic Imprints of both blocks B and C. Successively, we roughly cut the block C in two approximately similar blocks, namely blocks D (roughly $0.17 \times 0.082 \times 0.072$ m, 1320.5 g) and E (roughly $0.17 \times 0.084 \times 0.072$ m, 1486.4 g) and we measured their Sonic Imprints. Finally, we roughly cut the block E in two approximately similar blocks, namely blocks F (roughly $0.065 \times 0.082 \times 0.072$ m, 762.15 g) and G (roughly $0.061 \times 0.084 \times 0.072$ m, 722.1 g) and we measured their Sonic Imprints. Therefore, roughly speaking, we can affirm that the maximum dimensions of the investigated blocks were approximately 0.34 m (A), 0.17 m (blocks B, C, D and E), 0.083 m (blocks F and G). The resulting Sonic Imprints were not easy to be detected, especially for the smaller samples, due to two main reasons. The first is connected with the dimensions and weight of sensors (about 1.2 g) and the joining elastic strips (about 2 g), no longer insignificant with respect to the total weight of the samples, what's more added in the external part (that is with larger moment of inertia). The second reason was due to the available instruments (the ABEM seismometer or the prototype for artefacts of medium-large size of Fig. 1), which are really well working when the resonance frequencies are included in the range 0-6 kHz.

In addition, the environmental induced noise is obviously larger in the smaller and lighter pieces, giving rise to a reduction of the signal to noise ratio, especially at lower frequencies. For these reasons the measures have been repeated many times, anyhow giving not very clear results.



Fig. (6). Some phases of the acquisition of the Sonic Imprint

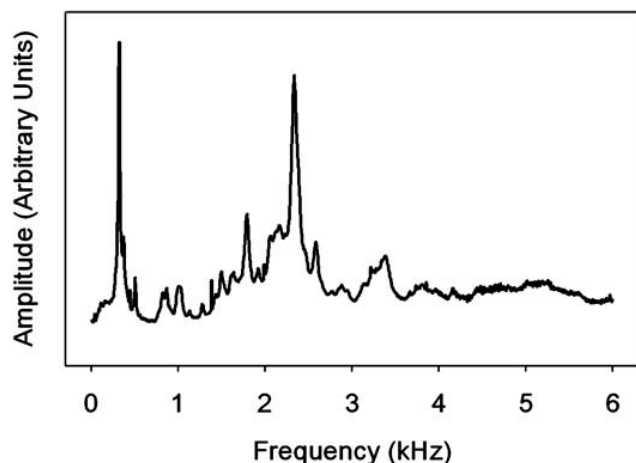


Fig. (7). The average spectrum of damped vibrations.

The average spectra of the recorded Sonic Imprints are presented in Fig. (10).

The difficulties to understand these results arise from the fact that the instrument now available, which has been made for large-medium pieces, is composed by elements (energizing source, probes, amplification) working in the range 0-6 kHz but having in the range 5-6 kHz the beginning of the response cutoff. But the main frequencies of the smallest block should be included in, or over, that frequency range. Blocks B, C D and E, that have main frequencies in the range between 2.5 and 4 kHz, also present the low-frequency range (marked with a broken line) still rich, but very unstable from a probe to another. This is probably due to the decreased distances between hitting points and detectors, with consequent larger sensitivity of some detectors.

5. CONCLUSIVE REMARKS

Finally, after many test and repeated controls, we can assess the performance of such methodology as follows.

- Acquisition of experimental data is absolutely non invasive and can be done in situ with a simple and light instrument, which is rather cheap.
- Acquisition of experimental data is fast (1/2 hour) and data processing is automatic (few minutes).
- Sonic Imprint identifies the artefact and depends on its geometry, the constituting material (elastic parameters and density) as well as on its internal defects. Sonic Imprint is a robust parameter, which is representative of the whole artefact.
- It is practically impossible to build an artefact having a given Sonic Imprint. Clones are easily recognizable.

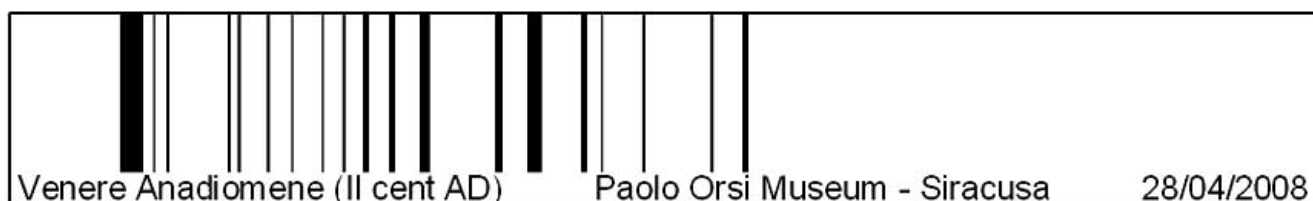


Fig. (8). The resulting bar code of the Sonic Imprint.

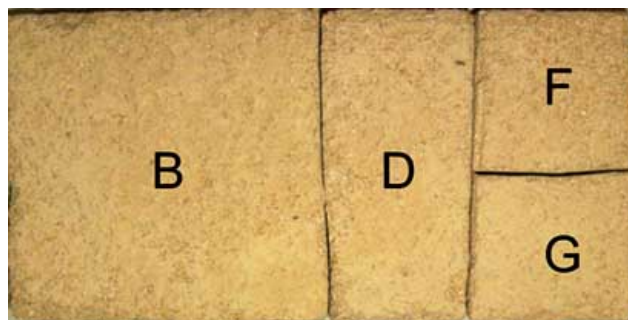


Fig. (9). The whole block A, divided in two blocks (B and C). Block C has been divided in two blocks (D and E) and E has been divided in two blocks (F and G).

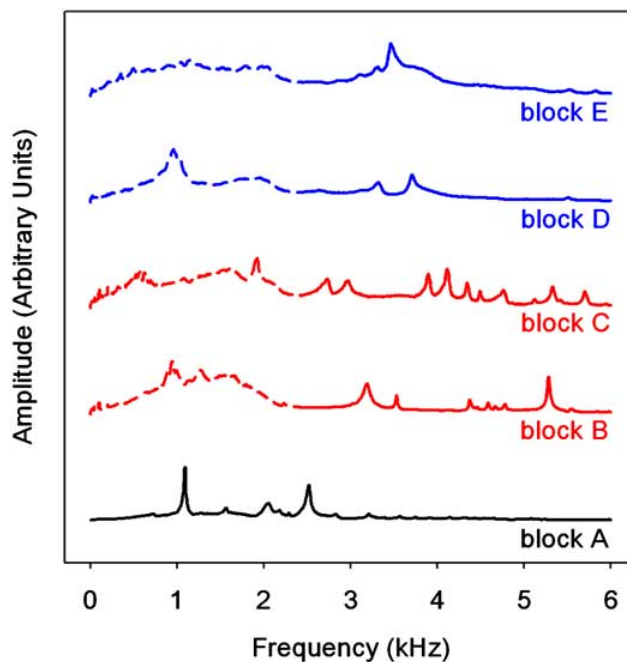


Fig. (10). The different sonic responses of some calcarenite blocks. The smallest blocks (F and G) gave responses containing only noise and are not presented here.

- Sonic Imprint of an artwork should change very slowly in time, except for severe and fast decay phenomena (e.g., cracks, imbibitions, etc.) modifying geometrical and/or elastic characteristics of the artefacts.
- Sonic Imprint can also be used to monitor the decay of an artwork.
- Tests regarding the application of this methodology to small artefacts require a number of adjustments in order to be fulfilled without problems. In particular, it is necessary to

build a new instrument working in the range of frequencies up to 30-40 kHz, i.e. up to the beginning of the low ultrasonic band. The probes should be selected not only to work with linearity in this range, but also very light in order to not affect the mass distribution of the tested artefacts. For this reason we are now testing some LP cartridges, able to detect vibrations in this whole range of frequencies. The laser displacement detectors, which seem to be very suitable to be used as probes because they do not need contact with the artefacts, up to date are able to work up to about 20 kHz. We hope that future technology can make available such sensors working up to higher frequencies.

Finally, a new tool should be prepared aimed to excite the damped vibrations of the artefacts. First tests carried out with a power tweeter seem to give interesting results.

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