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Combination of 1D laterally constrained inversion and 2D smooth inversion of resistivity data with a priori data from boreholes

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ABSTRACT
Resistivity imaging in combination with borehole information is a powerful tool for site investigation. We show that the combination of 1D laterally constrained inversion (1D-LCI) with the use of *a priori* information from borehole data and 2D smooth inversion adds significant value to the interpretation of continuous vertical electrical sounding (CVES) data. The 1D-LCI offers an analysis of the resolution of the model parameters. This is helpful when evaluating the integrity of the model. Furthermore, with the 1D-LCI it is possible to constrain model parameters with *a priori* information, e.g. depth-to-layer interfaces, based on borehole information.

We show that 2D smooth inversion resolves lateral changes well, while 1D-LCI results in well-defined horizontal layer interfaces. In geological environments where the lateral variations are not too pronounced, the 1D-LCI contributes to a geological interpretation of the resistivity measurements. Depths to layers can be interpreted with greater certainty than if using results from 2D smooth inversion only. The inclusion of *a priori* information in the inversion reveals further details and enhances the geological interpretation significantly.

INTRODUCTION

Resistivity imaging in combination with borehole data is a powerful tool in site investigation, and the development of efficient continuous vertical electrical sounding (CVES) systems (e.g. Griffiths and Turnbull 1985; van Overmeeren and Ritsema 1988; Dahlin 1996) has made the method one of the most frequently used in near-surface geophysics. CVES data are often inverted using 2D smooth inversion algorithms as presented by, for example, Oldenburg and Li (1994) or Loke and Barker (1996). An algorithm for multidimensional smooth inversion of resistivity data was presented by Pain *et al*. (2002). Olayinka and Yaramanci (2000) discussed the difference and compared 2D smooth inversion with 2D block inversion.

Nowadays, 2D smooth inversion is a standard procedure for inversion of CVES data. Pellerin (2002) has given numerous examples of the application, in engineering as well as for other purposes, where resistivity data have been inverted using 1D inversion, 1D-LCI or 2D smooth inversion. Applications of CVES measurements inverted with 2D smooth inversion have been described by Dahlin *et al*. (1999), Tirén *et al*. (2001) and Vickery and Hobbs (2003).

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A severe limitation of 2D smooth inversion is its inherent inability to determine sharp layer interfaces. This is to some extent improved by using the so-called robust inversion (Loke *et al*. 2003), where the misfit is minimized using the L_1-norm (e.g. Claerbout and Muir 1973). The block inversion discussed by Olayinka and Yaramanci (2000) also produces sharp layer interfaces. Tests with field data show that accurate results can be obtained for a two-layer model; however, when the subsurface is more complicated, the results can be unstable (Olayinka and Yaramanci 2000). Furthermore, the 2D block inversion is not accompanied by a full sensitivity analysis.

To produce sharp layer interfaces, we use the laterally constrained inversion (LCI) approach (Auken and Christiansen 2004). In this case, a 1D-LCI was used. The 1D-LCI performs 1D parametrized inversion of many separate models and data sets where neighbouring models are tied together with lateral constraints on the model parameters. The method is robust to the starting model (Auken and Christiansen 2004) and outputs a pseudo-2D lateral smooth model section with sharp layer interfaces. The 1D-LCI offers a sensitivity analysis of the model parameters. The sensitivity analysis is useful for evaluating the maximum number of layers that the resistivity data can resolve and for evaluating the integrity of the model. Furthermore, it is possible to add *a priori* information by constraining model parameters, e.g. depth-to-layer interfaces, based on lithology from borehole data. The 1D-LCI will be affected by 2D structures, limiting the applicability to geological environments with mainly horizontal features.

We show that a combination of 1D-LCI and traditional 2D-smooth inversion is a powerful tool for detecting various geological features. We compare 1D-LCI and 2D smooth inversion of high-density CVES data. The comparison is based on field data from a large CVES data set and on synthetic models reconstructing important structures in the field data. The field data are from Lockarp, Sweden, and were collected as part of a site investigation for The City Tunnel Project in Malmö (Bjelm and Wisén 2000). Extensive geological and geotechnical data from auger- and core-drilling are available as reference data. These reference data were used in three stages: firstly, for verification of the inversion results; secondly, as *a priori* information in the 1D-LCI; and thirdly, in combination with the resistivity models in the final geological interpretations.

GEOLOGY OF THE FIELD AREA

Lockarp is situated south of Malmö in south-west Sweden (Fig. 1). The geological environment in the Malmö area is sedimentary and consists of Quaternary deposits underlain by Danian limestone; a generalized profile is shown in Fig. 2. During different periods of the last glaciations, several icefronts moving in different directions have influenced the geological environment in the area. This has resulted in occasionally extreme geological variations.

The Quaternary deposits consist of four different units. From the surface and downwards they are:

Unit 1 – Post- or late-glacial sediments, consisting mainly of sand and silt. Since this layer is situated above the primary groundwater surface, these sediments can be dry, but they can also be semi-dry due to secondary aquifers, and/or mixed with the underlying clay till or organic material. The resistivity of this layer can vary significantly from about 100 Ωm to 1000 Ωm. The thickness is 0.5–2 m.

Unit 2 – Clay till, alternating with thin sand and silt layers. The resistivity of the clay till is typically between 20 and 100 Ωm. The thickness is 2–5 m.

Unit 3 – Sediments deposited between two clay tills, referred to as the inter-morainic sediments. This unit is found only in parts of the area. The inter-morainic sediments have been deposited on top of the lower clay till (i.e. Unit 4) and consist mainly of sand and silt layers. Sometimes the sediments contain layers of gravel or clay. The inter-morainic sediments are situated below the primary groundwater surface. The resistivity varies between 50 Ωm and 400 Ωm. The thickness can vary rapidly from 0 m to 3 m.

Unit 4 – Clay till containing silt and often sand. The resistivity of this clay till is slightly lower than the resistivity of the clay till in Unit 2, i.e. between 20 Ωm and 75 Ωm. The thickness ranges from 2 m to 10 m.

Unit 5 – Limestone. In the field area the limestone undulates slightly and rises about 10 m from the western to the eastern region. The top of the limestone is often crushed and mixed with the lower clay till (i.e. Unit 4). Sandy or very coarse local tills can be found directly on top of the limestone. The resistivity of the upper part of the limestone and the coarse local tills varies between 100 Ωm and 600 Ωm.

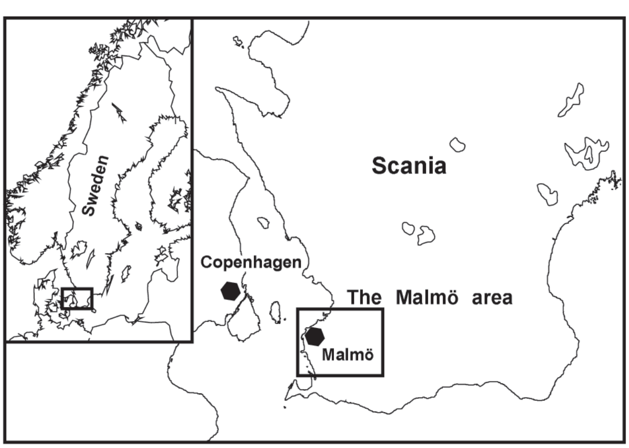

FIGURE 1
Map of the south of Sweden and the Malmö area. The construction area, the area in the box, is shown as an insert in Fig. 6.

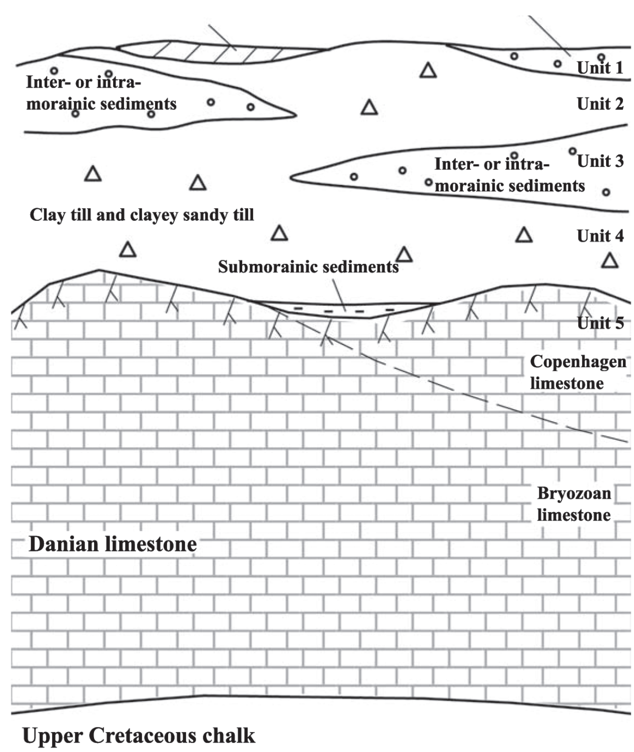

FIGURE 2
Generalized geological profile from the Malmö area. The Quaternary deposits are divided into four units underlain by Danian limestone.

METHODS

Laterally constrained inversion (LCI)

The 1D-LCI was originally developed for inverting pulled array continuous electrical sounding (PACES) data (Sørensen 1996). An extremely large quantity of data is obtained by the PACES system, and 2D smooth inversion is therefore not practical on a routine basis. Because the PACES system is used in the Danish sedimentary geological environments with relatively smooth lateral resistivity variations, a layered inversion model is desirable.

The 1D-LCI solves a number of 1D problems simultaneously with constraints between neighbouring models. This requires that all separate 1D models have the same subset of model parameters. The 1D-LCI approach is illustrated in Fig. 3(b). The CVES data set in Fig. 3(a) is divided into soundings and models. All models and corresponding data sets are inverted simultaneously, minimizing a common object function (Auken and Christiansen 2004). The lateral constraints and the constraints from *a priori* information are all part of the data vector, together with the apparent-resistivity data. Due to the lateral constraints, information from one model will spread to neighbouring models. If the model parameters of a specific model are better resolved, due to, for example, *a priori* information, this information will also spread to neighbouring models.

2D smooth inversion

For the 2D smooth inversion, the Res2DInv software (Loke and Dahlin 2002) was used. This software is commercially available. In this software it is possible to alter different inversion settings. The vertical-to-horizontal filter weight was set at 0.25, which enhances the horizontal features. Otherwise, default settings were used.

Combining LCI and 2D smooth inversion

Some of the considerations on which the 1D-LCI formalism is based coincide with the characteristics that make 1D-LCI a useful complement to 2D smooth inversion. Firstly, the inversion scheme is fast and capable of handling large data sets. Secondly, there is the possibility of combining different data types, e.g. transient electromagnetic data and DC data, and of including geological *a priori* information. Thirdly, the output model is accompanied by a sensitivity analysis of the model parameters. This provides a good quality control of the inverted models and enhances the subsequent geological interpretation.

The combination of 1D-LCI and 2D smooth inversion is expected to result in a better resolved geological model with direct influence from the available borehole data.

Lateral constraints

The lateral constraints can be considered as *a priori* information on the geological variability within the area where the measurements are taken. The smaller the expected variation for a model parameter is, the harder the constraint.

For the geology in the Lockarp area, lateral constraints on depths rather than on thicknesses are advantageous, due to the fact that layer interfaces rather than thicknesses are continuous. Furthermore, constraints on thicknesses do not relate to the actual depth of a layer, and an error in the thickness of a shallow layer will therefore affect all subsequent layers. For these reasons, constraints on depths have been used. The implementation

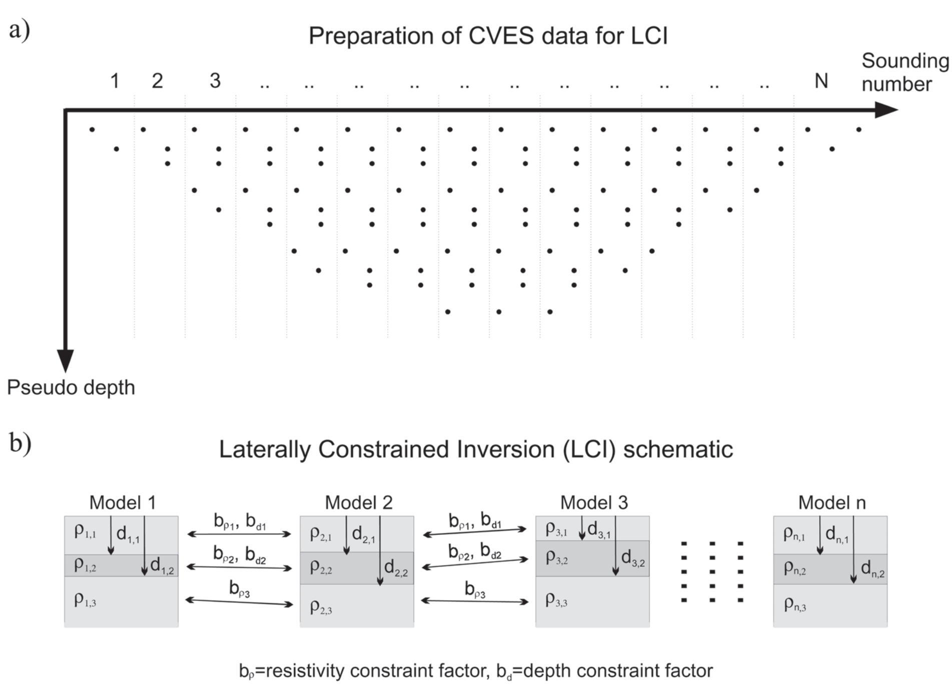

FIGURE 3
(a) The CVES profile is divided into *N* data sets. (b) The data sets are inverted simultaneously with a 1D model, resulting in a pseudo-2D image. The models are constrained laterally, and each model allows *a priori* constraints on the resistivities and thicknesses or depths.

of constraints on depths in the LCI algorithm is comprehensively described by Auken and Christiansen (2004).

A priori information

A priori information is added to the data set as depth to layers. The information originates from auger- and core-drillings. The *a priori* data are part of the total data vector together with the resistivity data.

If the *a priori* data agrees with the resistivity data, the depth to layers in the resistivity model will coincide with the *a priori* data. If, on the other hand, sufficient resistivity data suggest a different depth from that of the *a priori* data, the layer boundary in the resistivity model will probably not agree with the *a priori* data.

The 2D smooth inversion algorithm allows the inclusion of *a priori* information in fixed regions or layer boundaries. For the first option, the resistivity is constrained in a region, whereas for the second option, the smoothness constraints are loosened along layer boundaries. One single depth to a layer cannot be used to create such a region or layer boundary. If several data describing a layer boundary are available, these could be used, but interpolation between the data is then required. Due to the rapid horizontal changes in some of the geological units present in Lockarp, interpolation of geological units between boreholes is not possible. For example, if refraction seismic data were available, they could be used to make a better estimate of the depth-to-layer boundaries, but in this case no such data exist. Therefore, we will not try to constrain the 2D smooth inversion.

Quality control of the inverted model

The sensitivity analysis of model parameters from 1D-LCI is used to assess the resolution of the inverted model. The parameter sensitivity is the linearized approximation to the covariance of the estimation error (Auken and Christiansen 2004).

The analysis of a specific parameter will be characterized by a standard deviation factor (*STDF*). The case of perfect resolution has *STDF*=1, well-resolved parameters have *STDF*<1.3, moderately resolved parameters have *STDF*<1.5, poorly resolved parameters have *STDF*<2, and all model parameters having *STDF*>2 are considered to be unresolved (Auken and Christiansen 2004).

For 2D smooth inversion, the root-mean-square (rms) error between observed resistivity and model response is used to verify the quality of the inversion. This measure has also been used here for 1D-LCI. In neither LCI nor 2D smooth inversion does the rms error consider any of the constraints optimized. The rms error was calculated as

$$RMS\ error = \left[\frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{R_{obs} - R_{est}}{R_{obs}} \right)^2 \right]^{\frac{1}{2}} ,$$

where R_{obs} is the observed resistivity, R_{est} is the estimated resistivity model response, and N_d is the total number of data.

INVERSION OF SYNTHETIC DATA

In order to investigate the behaviour of the two inversion algorithms, a suite of synthetic data sets was generated and subsequently inverted and compared. The models are mainly based on the geological settings in the field area. In order to test the practicability of *a priori* information, we have simulated two boreholes and used these as *a priori* information in the 1D-LCI. The information obtained from the simulated boreholes consists of depth-to-layer interfaces. Furthermore, optimal inversion parameters for the 1D-LCI were determined.

For the forward modelling, we used the Wenner configuration with electrode spacings from 2.0 m to 48.0 m. The forward modelling was carried out using the finite-element program, Res2DMod (Dahlin and Loke 1998). No Gaussian noise was added to the data. However, the results from this study are clear and will probably be the same with only a few percent of noise added to the data.

A factor of 1.12, or approximately 12% of the absolute parameter value, is a reasonable value for the vertical and horizontal constraints on resistivities and depths (Foged 2001). The lateral constraint for the depth to the bottom layer was, in this case, set at 1.06. This was done for two reasons: firstly, to adjust for the fact that these constraints are relative, the absolute constraint otherwise being too big for the bottom layer, and secondly, because the bottom layer, i.e. the depth to the limestone surface, is expected to have the smallest variation of all the layer boundaries.

Results of synthetic inversion studies

This presentation is limited to one of the synthetic models. The model has five layers as shown in Fig. 4(a). A slight change in resistivity in the third layer (Unit 3 representing the intermorainic sediments) simulates a change in composition of the sediments. Data from forward modelling are presented in a pseudosection in Fig. 4(b). Five layers were used in the 1D-LCI models. The number of layers in 1D-LCI is based on the results from 2D smooth inversion and on the sensitivity analysis of model parameters in 1D-LCI. The sensitivity analysis will show if all model parameters are resolved or if the model should be reduced.

The results clearly demonstrate some of the advantages and disadvantages of the different inversion algorithms. The 2D smooth inversion recovers the small lateral change in resistivity in the third layer (Fig. 4c,d). This change is not captured by the 1D-LCI (Fig. 4e,f). The horizontal layer interfaces are clearly described by the 1D-LCI. These interfaces are not clear after 2D smooth inversion, particularly in the L_1-norm section (Fig. 4d). When *a priori* information is used in the 1D-LCI, the correct levels are mapped. The 2D smooth inversion shows a depression of the level and the resistivity of the bottom layer in the central parts of the section, below the high-resistivity part of the third layer. The inversion using the L_1-norm does not recover the model beneath this part at all. This depression is caused by high-resistivity equivalence.

The sensitivity analysis of model parameters from 1D-LCI

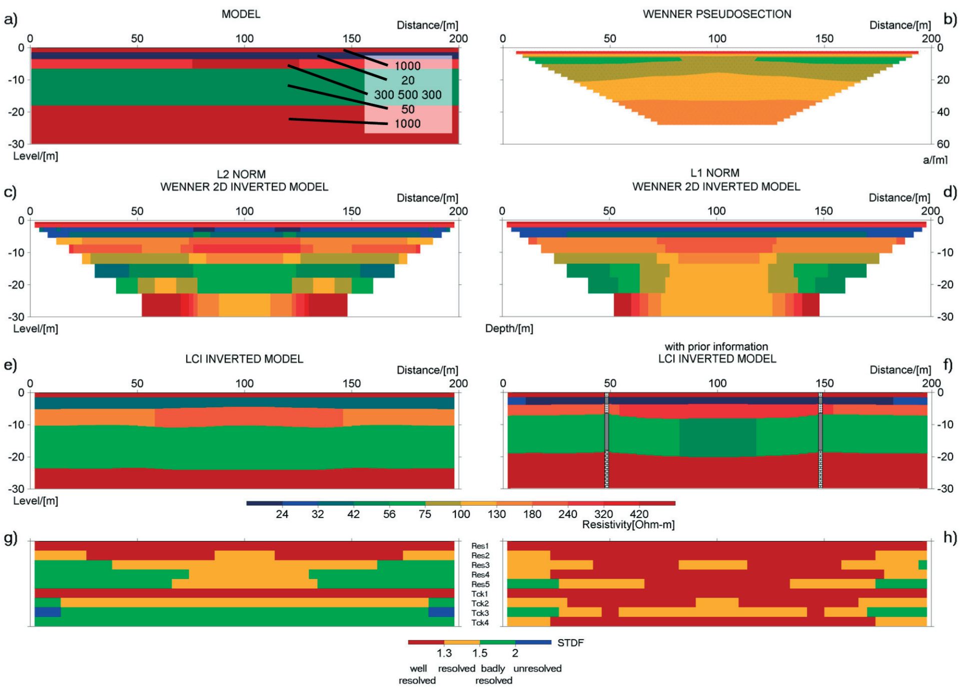

FIGURE 4
(a) True model for resistivity forward modelling. (b) Data presented as a pseudosection. (c) Inversion result from Res2Dinv using L_2-norm (rms error: 0.44%) and (d) L_1-norm (rms error: 0.59%). (e) Inversion results from 1D-LCI using 5 layers without *a priori* information (rms error: 0.37%) and (f) with *a priori* information (rms error: 0.38%). (g) Analysis of model parameters for 1D-LCI without *a priori* information and (h) with *a priori* information.

shows that the resistivity of layers 1 and 2 and the thickness of layer 1 are mainly well resolved. The rest of the model parameters are moderately or badly resolved. When *a priori* information is added to the data, the resolution of model parameters is mainly well resolved. The resistivities of the third and the fifth layers and thickness of the third layer are moderately resolved. When the analysis of the 1D-LCI section with *a priori* information is studied on a more detailed *STDF* scale (Fig. 5), it can be seen that the model parameters, as expected, are better resolved around the models where the *a priori* information was added, and that this effect spreads sideways through the lateral constraints.

Discussion of synthetic results

The 2D modelling results in well-defined horizontal changes while inversion with the 1D-LCI modelling gives well-defined horizontal layer interfaces. The analyses of model parameters from 1D-LCI show that the model parameters are better resolved when *a priori* information is added. The model resolution is improved around the added *a priori* information, and this positive effect spreads to the neighbouring models through the lateral constraints. The third and fifth units are more difficult to resolve than the other parameters, but it is clear that high-resistivity equivalence, to a large extent, is resolved when *a priori* information is added to the inversion.

CASE STUDY: MALMÖ CITY TUNNEL PROJECT – LOCKARP

At this specific location, a railway trench of about 2 km length and 10 m depth will be excavated. The main issues in the site investigation are the mapping of the thickness of the Quaternary deposits (Units 1–4) and the extent of the inter-morainic sediments (Unit 3). Because these units control the water inflow to the trench, their exact extent is important for the trench construction.

In total, about 3 km of CVES resistivity measurements were performed. One profile following the planned position of the railway is presented here. The location of the measurements is shown in Fig. 6. The eastern half of the section was acquired using a combination of Wenner and Schlumberger electrode configurations. In this area, the data density is equal to that in the synthetic data sets. The western half of the section was acquired using a Wenner configuration only, and contains about one-third of the data in the eastern part.

The resistivity data were originally acquired for 2D inversion. For 1D-LCI, the data were divided into individual soundings at distances of 4 m. The complete data set for 1D-LCI contains exactly the same amount of data as the data set for 2D smooth inversion. Inversion performed with 1D-LCI and 2D smooth inversion using the L_2-norm are presented.

FIGURE 5
Detailed plot of the analysis of model parameters from 1D-LCI.

Results of field data inversion

Figure 7 shows the pseudosection, inverted models and sensitivity analysis from an almost 2 km long section. Five layers were used in the 1D-LCI section. The choice of this number of layers was based on three factors: the sensitivity analysis of model parameters in 1D-LCI, the result from 2D smooth inversion and the number of geological units in the generalized geological profile.

All the inverted models (Fig. 7b,c,d) have a continuous high-resistivity layer at the bottom. Above this layer is a low-resistivity layer, sometimes divided by a discontinuous high-resistivity layer. On top is a thin high-resistivity layer in parts of the profile.

It is evident that the high-resistivity layer at the bottom corresponds to geological Unit 5 (the limestone). The low-resistivity layer corresponds to Units 2 and 4 (the clay till). The discontinuous high-resistivity layer within the clay till corresponds to Unit 3 (the inter-morainic sediments). The high-resistivity layer in the top corresponds to Unit 1 (the post-glacial sediments).

The 2D smooth resistivity model (Fig. 7b) has high resolution in the horizontal direction. It clearly indicates where the inter-morainic (Unit 3) and post-glacial (Unit 1) sediments are present. It describes the top and bottom of the inter-morainic sediment, but, due to the smoothness in the model, it is not pos-

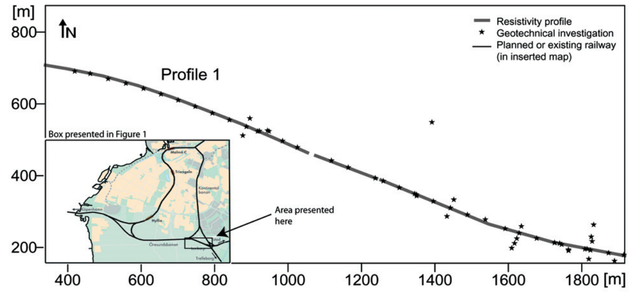

FIGURE 6
Map of Lockarp showing the investigated sections (thick black lines) and the existing railways (dashed lines). The position of the planned railway trench coincides with the longest section.

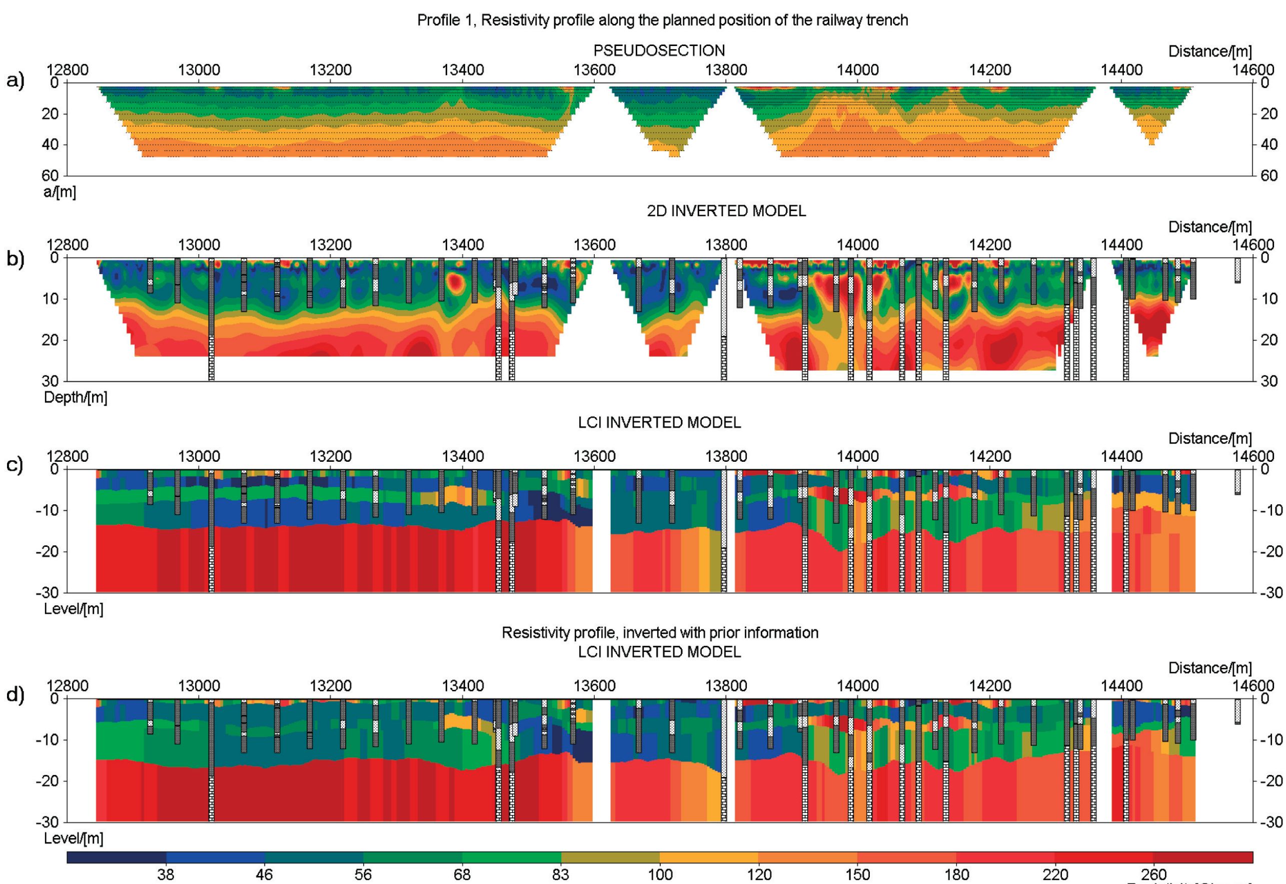

FIGURE 7
Inversion result from a profile following the planned position of the railway trench. (a) Data presented as pseudosection. (b) Result from L_2-norm 2D smooth inversion using Res2Dinv (rms error: 2.26%). (c) Result from 1D-LCI with 5 layers (rms error: 3.89%). (d) result from 1D-LCI with 5 layers inverted with *a priori* information (rms error: 3.93%).

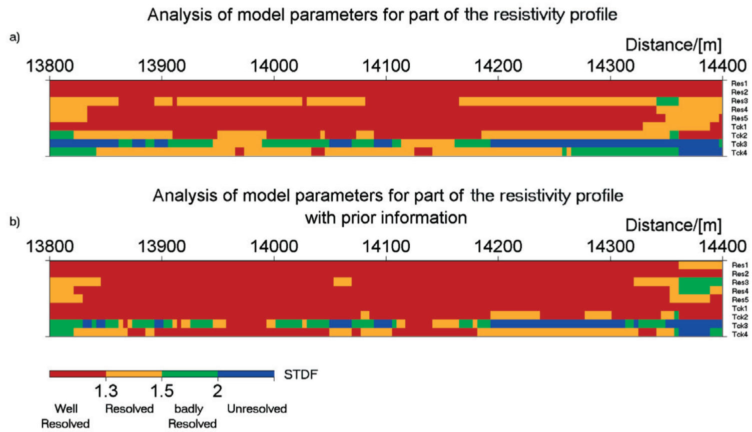

FIGURE 8
Analyses of model parameters from LCI of part of profile 1: (a) without *a priori* information; (b) with *a priori* information.

sible to detect sharp boundary interfaces. Furthermore, the level of the lower interface does not agree with the borehole data. High-resistivity equivalence due to the presence of inter-morainic sediment (Unit 3) results in an apparent depression in the level of the limestone and its resistivity. It is generally very difficult to determine a distinct boundary for the top of the limestone, and below the inter-morainic sediments it is impossible.

The 1D-LCI model (Fig. 7c) clearly describes the horizontal layer interfaces of the different geological units. The inter-morainic sediments are easily identified, but their lateral extension is difficult to interpret due to the horizontally smeared model. The sensitivity analysis of model parameters (Fig. 8a) shows that the model is mainly well resolved. However, the thickness of the third layer, the inter-morainic sediments (Unit 3), is almost unresolved.

The 1D-LCI model with *a priori* information added to the inversion (Fig. 7d) is overall better resolved (Fig. 8b). The sensitivity analysis shows that the third layer is slightly better resolved. There is a higher contrast between the high- and the low-resistivity geological units. The effect of adding *a priori* information is clearly visible. In the first half of the profile, profile coordinates 12850 – 13450 m, the level of the fifth layer, the limestone, shifts downwards several metres.

DISCUSSION

Effects of using *a priori* information

In the case study presented here, a large amount of high-quality borehole data is used as *a priori* information. The *a priori* information is especially important as it helps to resolve high-resistivity equivalences. The addition of *a priori* information causes changes in the level of layer interfaces in the entire model. Although these changes are often small, they result in better-determined layer resistivities. Examples of this appear between profile coordinates 13850 m and 14200 m, where thickness and resistivity change, due to the *a priori* information.

In some positions, it is clear that the borehole data differ from the resistivity data, e.g. for the depth to layer 5 at position 13010 m and around 13500 m in Fig. 7(c). In these cases, a discrepancy between the measured apparent resistivity and the model response would increase the total residual error more than the discrepancy between *a priori* information and the resistivity model layer boundaries.

It is unusual to have such an extensive data set with borehole data. In order to determine the benefit obtained from the added *a priori* information, an inversion was performed with half the available *a priori* information. The part of the resistivity profile between coordinates 13800 m and 14400 m was used. The inversion results (Fig. 9) indicate that inversion with half the *a priori* information gives the same result as inversion with all available *a priori* information.

If optimally positioned, a much smaller amount of *a priori* information would probably be sufficient. This can be achieved by performing the resistivity measurements early on during the site investigation and then positioning the geotechnical investigations based on the interpreted resistivity models. The increased geological knowledge acquired before positioning of expensive drillings will result in a more cost-effective site investigation with a more reliable final geological model.

Combining 1D-LCI and 2D smooth inversion

The final geological interpretation should be made based on the 2D smooth inversion section and the 1D-LCI section with *a priori* information. In this case, a better geological interpretation can be made than that from any of the three data sets alone. When the *a priori* information is in agreement with the resistivity data, the total residual error from 1D-LCI should not increase significantly compared to the result from 1D-LCI without *a priori* information.

CONCLUSIONS

Comparing results from the synthetic models, we conclude that 2D inversion shows good horizontal resolution, but insufficient vertical resolution. The 1D-LCI yields well-defined horizontal layer interfaces. When *a priori* information is added, the resolution of the model parameters is improved, and high-resistivity equivalence is resolved. The positive effect due to *a priori* infor-

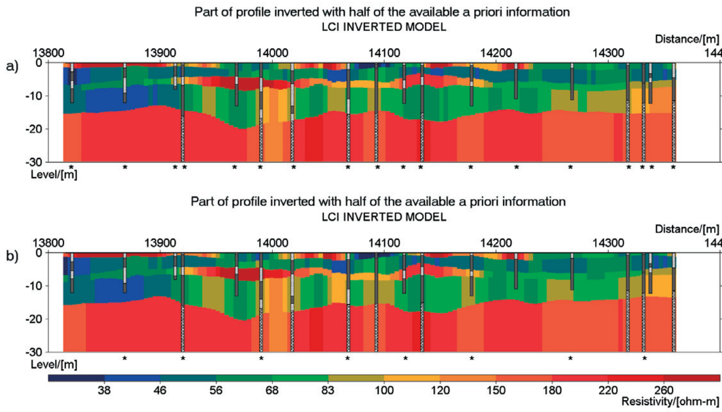

FIGURE 9
Inversion result from evaluation of the effect of *a priori* information. (a) Result from 1D-LCI with 5 layers inverted with all available *a priori* information (rms error: 4.0%); (b) result from 1D-LCI with 5 layers inverted with half of the available *a priori* information (rms error: 4.0%).

mation added to one model spreads sideways through the lateral constraints to the neighbouring models. The sensitivity analysis of model parameters is important in the quality control of the resistivity model.

For inversion of field data from Lockarp, the results from 2D inversion show good horizontal resolution but they do not define the depth-to-layer interfaces accurately. 1D-LCI gives distinct layer interfaces. Addition of *a priori* information results in a better resolved and more reliable resistivity model. As in the case of synthetic modelling, *a priori* information improves the inversion result when used to solve ambiguity due to high-resistivity equivalence.

It can be concluded that inversion results from the 2D smooth inversion and the 1D-LCI complement each other very well. Together with geotechnical and/or geological reference data, the joint interpretation of resistivity sections from these two methods increases the possibilities of a precise geological interpretation.

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REFERENCES

Auken E. and Christiansen A.V. 2004. Layered and laterally constrained 2D inversion of resistivity data. *Geophysics* **69**, 752–761.

Bjelm L. and Wisén R. 2000. *Part of site investigation at Lockarp* (in Swedish). Internal report for the City Tunnel Project (CTP), Lund, Sweden.

Claerbout J.F. and Muir F. 1973. Robust modelling with erratic data. *Geophysics* **38**, 826–844.

Dahlin T. 1996. 2D resistivity surveying for environmental and engineering applications. *First Break* **14**, 275–283.

Dahlin T., Bjelm L. and Svensson C. 1999. Use of electrical imaging in site investigations for a railway tunnel through the Hallandsås Horst, Sweden. *Quarterly Journal of Engineering Geology* **32**, 163–172.

Dahlin T. and Loke M.H. 1998. Resolution of 2D Wenner resistivity imaging as assessed by numerical modelling. *Journal of Applied Geophysics* **38**, 237–249.

Foged N. 2001. *Inversion of 2-dimensional resistivity distributions using laterally constrained models* (in Danish: *Inversion med lateralt sammenbundne modeller af 2-dimensionale stokastiske resistivitets-fordelinger*). Thesis, University of Aarhus, Denmark.

Griffiths D.H. and Turnbull J. 1985. A multi-electrode array for resistivity surveying. *First Break* **3**(7), 16–20.

Loke M.H. and Barker R.D. 1996. Rapid least-squares inversion of apparent resistivity pseudosections by quasi-Newton method. *Geophysical Prospecting* **44**, 131–152.

Loke M.H. and Dahlin T. 2002. A comparison of the Gauss-Newton and quasi-Newton methods in resistivity imaging inversion. *Journal of Applied Geophysics* **49**, 149–162.

Loke M.H., Dahlin T. and Acworth I. 2003. A comparison of smooth and blocky inversion methods in 2-D electrical imaging surveys. *Exploration Geophysics* **34,** 182–187.

Olayinka A.I. and Yaramanci U. 2000. Use of block inversion in the 2-D interpretation of apparent resistivity data and its comparison with smooth inversion. *Journal of Applied Geophysics* **45**, 63–81.

Oldenburg D.W. and Li Y. 1994. Inversion of induced polarization data. *Geophysics* **59**, 1327–1341.

van Overmeeren R.A. and Ritsema I.L. 1988. Continuous vertical electrical sounding. *First Break* **6**, 313–324.

Pain C.C., Herwanger J.V., Worthington M.H. and de Oliveira C.R.E. 2002. Effective multidimensional resistivity inversion using finite-element techniques. *Geophysical Journal International* **151**, 710–728.

Pellerin L. 2002. Applications of electrical and electromagnetic methods for environmental and geotechnical investigations. *Surveys in Geophysics* **23**, 101–132.

Sørensen K.I. 1996. Pulled Array Continuous Electrical Profiling. *First Break* **14**, 85–90.

Tirén S.A., Wänstedt T. and Sträng T. 2001. Moredalen - a canyon in the Fennoscandian Shield and its implication on site selection for radioactive waste disposal in south-eastern Sweden. *Engineering Geology* **61**, 9–118.

Vickery A. and Hobbs B.A. 2003. Resistivity imaging to determine clay cover and permeable units at an ex-industrial site. *Near Surface Geophysics* **1**, 21–30.