Modeling induced polarization effects in helicopter time domain electromagnetic data: Synthetic case studies

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ABSTRACT

We have developed a synthetic multiparametric modeling and inversion exercise undertaken to study the robustness of inverting airborne time-domain electromagnetic (TDEM) data to extract Cole-Cole parameters. The following issues were addressed: non-uniqueness, ill posedness, dependency on manual processing and the effect of constraints, and a priori information. We have used a 1D layered earth model approximation and lateral constraints. Synthetic simulations were performed for several models and the corresponding Cole-Cole parameters. The possibility to recover these models by means of laterally constrained multiparametric inversion was evaluated, including recovery of chargeability distributions from shallow and deep targets based on analysis of induced polarization (IP) effects, simulated in airborne TDEM data.

Different scenarios were studied, including chargeable targets associated with the conductive and resistive environments. In particular, four generic models were considered for the exercise: a sulfide model, a kimberlite model, and two generic models focusing on the depth of investigation. Our study indicated that, in cases when relaxation time ($\tau$) values are in the range to which the airborne electromagnetic is most sensitive (e.g., approximately 1 ms), it is possible to recover deep chargeable targets (to depths more than 130 m) in association with high electrical conductivity and resistive environments. Furthermore, it was found that the recovery of a deep conductor, masked by a shallower chargeable target, became possible only when full Cole-Cole modeling was used in the inversion. Lateral constraints improved the recoverability of model parameters. Finally, modeling IP effects increased the accuracy of recovered electrical resistivity models.

INTRODUCTION

The interest in the possibility of recovering induced polarization (IP) parameters from airborne time-domain electromagnetic (TDEM) data has been recently increased from the mineral exploration industry and airborne electromagnetic (AEM) data providers, after nearly two decades of scant publication on the subject (Flis et al., 1989; Smith, 1989; Smith and Klein, 1996; Raiche, 1998; Kratzer and Macnac, 2012; Viezzoli et al., 2013). The interest has increased to such degree that airborne-induced polarization (AIP) has been marketed as a tool for mineral exploration. Recent work by Macnac (2015) and Kang and Oldenburg (2015) testifies the renewed momentum in this field, also within the academic community.

The better signal-to-noise ratio available in the most recent larger dipole systems undoubtedly has led to the renewed attention in the airborne IP phenomenon. Arguably, more awareness and more rigorous processing from data providers (e.g., not deleting the negative secondary EM field voltage values) also revealed more “IP-like effects” in AEM data sets. The next logical step in the AIP legacy is advanced modeling and inversion of the AIP effects, for which several groups are contributing worldwide. This is the first of two papers dedicated to a more thorough study of AIP in TDEM data. It presents a series of quasi-2D synthetic forward and inverse modeling examples.

Among the several physical IP models reported in the literature (e.g., generalized effective-medium theory of induced polarization [GEMTIP] theory by Zhdanov, 2008), we use perhaps the most popular, i.e., the Cole and Cole (1942) model (Pelton et al., 1978). The objective is to model four Cole-Cole parameters ($\rho_0$, DC resistivity; $m_0$, chargeability; $\tau$, relaxation time; and $c$, frequency parameter) to comprehensively address the relevance of IP effects in heliborne TDEM data using various realistic scenarios. We further study the possibility to recover the Cole-Cole parameters by means of multi-parametric quasi-2D laterally constrained inversion (LCI). This study is the companion to another study (Kaminski and Viezzoli, 2016),...
which is dedicated to field case studies, based on Cole-Cole modeling and inversions (quasi-3D) of AEM data, collected for a range of mineral exploration targets. Together, the two papers provide a rigorous reference and insight on the AIP phenomenon and its potential relevance to exploration and geologic mapping in general.

This study is the natural continuation of previous work including Fiandaca et al. (2012), Kaminski et al. (2015), Viezzoli et al. (2013, 2015), and Viezzoli and Kaminski (2016). In this paper, a variety of 2D-like synthetic models are used to simulate versatile time domain electromagnetic (VTEM) full-waveform (Fiandaca et al., 2012) data sets that are contaminated with noise and then inverted to study the recoverability of various targets in various environments. The forward computation is 1D, and the synthetic models are therefore actually 2D-like sections obtained from stitched together 1D models. For the forward modeling, a typical 7.2 ms width VTEM current pulse waveform was used (Figure 1). The following four synthetic models were considered:

1) Disseminated sulfide model: In this model, two targets were introduced. First is a nonconductive and chargeable body (representing disseminated sulfide), whereas the second is a conductive and nonchargeable (representing a general conductor) (Figure 2a and 2b). The ability to recover the chargeable response from the resistive target was studied.

2) Kimberlite model: In this model, a synthetic kimberlite pipe was placed underneath 30 m overburden (Figure 3a and 3b). The upper facies of the kimberlite (crater) is chargeable and conductive, whereas the lower facies of the kimberlite (diatreme) is less conductive and nonchargeable. The recovery of depth to the chargeable target was studied.

3) Dipping chargeable layer model: In this model, a dipping conductive and chargeable target starts at surface and reaches a maximum depth of 150 m. The ability to recover deepening chargeable targets is studied in resistive and conductive nonchargeable host rock environments (Figure 4).

4) “Deep conductors” below chargeable layers model: This model is otherwise similar to model 3, but introduces a deep-buried (300 m) layer with varying conductivity, emplaced below a deepening chargeable layer (Figure 5). The masking of the deeper conductive bodies from the IP effect due to the shallower, conductive polarizable layer is studied, as well as the ability to recover the deep conductive layer by means of quasi-2D inversion (LCI).

**METHODOLOGY**

**Approach**

The goal of this paper is to provide insight into an approach for modeling IP in AEM data, which can be applied to large-scale field data sets. For inverting field data, we use the constrained inversion approach, either quasi-2D (LCI; Auken and Christiansen, 2004) or quasi-3D (spatially constrained inversion [SCI]; Viezzoli et al., 2008). It is based on an objective function with a 1D forward response and with either 2D or 3D constraints on the covariance of model parameters belonging to neighboring soundings. Such an
approach is applicable in cases with moderately dipping geologies and moderate contrast in electrical properties. In other cases, 2D/3D artifacts may be present in LCI/SCI inversions in the form of edge effects. When compared with the true 1D inversion (when each sounding is inverted individually, without any spatial covariance between neighboring stations), the advantages of the LCI/SCI inversion are the suppression of noise and the reduction of the non-uniqueness, which allow recovery of the lateral continuity of the inverted models.

Given that for the field data we use 1D forward response (Kaminski and Viezzoli, 2016), all of the synthetic studies subject of the current paper also adopt the same 1D approach. The forward responses and the inversions use the same 1D kernel (AarhusInv, Auken et al., 2014), ensuring that, although some 2D effects of the modeling are disregarded by the inversions, the system response is nonetheless properly modeled and therefore, the obtained results are reliable.

As mentioned earlier, we use the Cole-Cole model and invert for all its parameters at once (multiparametric inversion). A total of 101 synthetic stations were used for the 1D forward modeling (for each of the four synthetic models), with a separation of 30 m. A nominal flight elevation of 30 m over flat terrain was considered.

The forward response is calculated for resistivity only and for a full suite of Cole-Cole parameters. The VTEM transmitter waveform used for the simulation is shown in Figure 1, and the time gates used in the simulation are described in Table 1. The simulated data units are in volts (V) normalized by the receiver area (m$^2$). A current of 160 A is used for the modeling of VTEM system data, along with four turns in the transmitter loop. To bring the units of the modeled data to the standard VTEM measurement units of $\frac{pV}{(A \cdot m^4)}$, they need to be multiplied by the corresponding peak current, multiplied by the number of turns in transmitter loop, normalized by the transmitter area, and multiplied by $10^4$.

The synthetic data are further contaminated with noise (Munkholm and Auken, 1996) and inverted with different starting parameters and constraint types, including varying lateral constraints and additional regularization parameters as a priori information. Despite the fact that the synthetic forward modeling is carried out in 1D, it is still meaningful to invert the data using the LCI approach, taking into consideration the ill posedness of multiparametric inversion, which requires nontrivial regularization. A measure of depth of investigation (DOI) (Fiandaca et al., 2015) for each of the output model parameters is also calculated and used to mask the results. In addition, some noise-contaminated data were artificially processed, replicating the advanced processing techniques required for optimal results of field data AIP modeling (Kaminski and Viezzoli, 2016).

The objective of multiparametric inversion was to test a wide selection of starting models, as well as different types of constraints (vertical and horizontal) imposed upon the $\tau$ and $c$ parameters to test what role locking $\tau$ and $c$ plays in the ability to recover the true model.

Starting resistivity values are tested for half-spaces ranging from 10 to 1000 ohm-m; starting chargeability values are tested ranging from 10 to 100 mV/V; and the starting relaxation time constant ($\tau$) values are tested from $10^{-4}$ to $10^{-2}$ s (consistent with the expected range of sensitivity to this parameter in known airborne TDEM systems) (Macnae, 2015). Parameter $c$ is tested ranging from 0.3 to 0.7 values (Table 2).

These starting parameters yield 81 unique combinations for each synthetic data set. Two sets of constraints are then tested on $\tau$ and $c$. First, all 81 starting model combinations are associated to “soft” constraints (allowing $\tau$ and $c$ to vary up to 10% spatially between neighboring stations and between neighboring layers), then to “hard” constraints (locking the spatial variance between neighboring stations and layers in $\tau$ and $c$ to 0.1%). Therefore, a total of 162 combinations are obtained.

Furthermore, the measure of IP effect is quantified for the synthetic data, measured as a metric of the difference between IP and non-IP responses from the same model and representative of IP effect detectability in the noisy data (metric of the IP effect).

**Metric of the IP effect**

To better understand the extent of the IP effect, we introduce an IP effect metric defined as a weighed difference between transients affected by IP and those not affected by IP. The absolute values of these differences calculated for each datum are summed up on a

Figure 3. Synthetic quasi-2D kimberlite true model. (a) Resistivity model (top 150 m). (b) Chargeability model (top 150 m). (c) IP effect metric (blue curve), compared with system noise (red line).
transient to transient basis and plotted in logarithmic space. The logarithmic space approach reduces skewness of the metric toward greater absolute values in data space and therefore reduces the sensitivity of the metric to early times in the transients. Plotting the metric in logarithmic space also makes it consistent with similar noise-level considerations, as those in the predicted data.

Equation 1 describes the IP effect metric:

$$m_{IP} = \frac{\sum_{j=1}^{n_t} \log_{10} |V_{j,NOIP} - V_{j,IP}|}{n_t},$$

where $m_{IP}$ is the calculated IP effect metric; $V_{NOIP}$ is the recorded voltage, for the model without IP, at the $j$th time gate in the EM decay transient; $V_{IP}$ is the corresponding voltage with the consideration of the Cole-Cole model at the $j$th time gate; and $n_t$ is the number of time gates in the transient for which the measure is calculated. The normalization of the measurement by the number of time gates makes this metric invariant to the sampling rate and therefore uniform and applicable to any EM transient. The $m_{IP}$ metric is given in log$_{10}$ (volts), and it is directly comparable with the noise level of a given system. When the $m_{IP}$ amplitudes are larger than the noise levels, the IP effect can be measurable in the data space.

Figure 4. (Left) Synthetic quasi-2D simple deep reference model (fixed $\tau$ and $c$ parameters to $10^{-3}$ and 0.5, respectively); the top 150 m are shown. (Right) Simulated noise-free dBz/dt transients (randomly colored) for the profile distance interval between 0 and 1440 m. The deeper the chargeable anomaly, the later the time of the sign reversal. The dashed red line represents typical noise level of the system.

Figure 5. Synthetic quasi-2D deep conductor true model (fixed $\tau$ and $c$ parameters to $10^{-3}$ and 0.5, respectively); the top 400 m are shown.
Metric of the model norm

We also introduce and calculate a metric for each of the Cole-Cole model parameters to provide a quantitative assessment, beyond visual appearance, of the differences between the true and the recovered models. The matrix $\Delta \text{Par}_{i,j}^k$ is obtained as the difference between the value output from each of the inversions ($k$, with $k = 1, 162$, as per the total number of inversions tested for each data set) versus the true value, for each layer ($i$, with $i = 1, 29$) and for each station ($j$, with $j = 1, 101$). This metric can be plotted in a 2D section, showing spatially the discrepancies between the true and the output (out) models, for a given Cole-Cole parameter and the results of the inversions of a given combinations of starting models and constraints.

The differences are represented in the form of equation 2,

$$\Delta \text{Par}_{i,j}^k = \log_{10} \text{Par}_{i,j}^{\text{out}(k)} - \log_{10} \text{Par}_{i,j}^{\text{true}},$$

where $\Delta \text{Par}_{i,j}^k$ is the difference matrix and matrices $\text{Par}_{i,j}^{\text{true}}$ and $\text{Par}_{i,j}^{\text{out}(k)}$ are representative of values associated with the true and recovered models, respectively.

This measure provides representative graphical description of the discrepancies between the output of one specific inversion and the true model. It can be also used for statistical analysis of multiple inversions carried out with one starting parameter selection versus a similar suite of inversions, with another parameter selection (e.g., inversions with a locked $c$ parameter versus inversions with a constrained $c$ parameter).

Of the several possible simple statistical indicators, the mode is the most adequate because it calculates the most frequent value occurring in a group. We therefore calculate the 2D matrix mode($\Delta \text{Par}_{i,j}$) for each parameter, over the full suite of different inversions.

Synthetic data noise contamination

The standard deviation of noise for each gate ($\sigma_j$) takes into account a multiplicative contribution ($\sigma_0$), uniformly on all time gates (3% of the signal) and an additive contribution ($\sigma_{\text{noise}}$), resulting from a model noise that can be approximated to a straight line with a slope $r^{-1/2}$ in a log-log plot. The additive term represents the “white” noise, which can be reduced by adjusting the width of the gates and by stacking. From real VTEM data, we derived an estimate value for the white noise of $10^{-11.5}$ V/(Am$^2$), at 1 ms. The multiplicative term represents misalignment and geologic (physical property) complexity. In these synthetic exercises, we assume that the data are free from any other potential sources of noise such as bias at the early times or incorrect leveling of the late times that can affect actual AEM data. The total noise contribution to synthetic data can be described in equation 3:

$$\sigma_j = \sigma_0 r^{-1/2} + \sigma_{\text{noise}}.$$
\[ \sigma_i = \sqrt{\sigma_{0,i}^2 + \sigma_{\text{noise},i}^2}. \]  

(3)

The specific considerations on inversion methodologies customized for each of the following four synthetic models are explained in the corresponding sections below, as well as the results and their discussions.

**SYNTHETIC SULFIDE MODEL**

**True model and forward modeling**

The resistivity and chargeability true model are shown in Figure 2. As it can be seen from Figure 2a and 2b, the model consists of four general rock types: a slightly chargeable overburden (OB), a nonchargeable conductor (S), a chargeable and resistive disseminated sulfide (M), and the nonchargeable, resistive host rock (HR). The true Cole-Cole parameters used in the model are provided in Table 3.

For the disseminated sulfide, we have used \( m_0 = 350 \text{ mV/V}, \tau = 1 \text{ ms}, \) and \( c = 0.5. \) According to Pelton et al. (1978), who measure (galvanically) several samples from different mineralizations around the world, these parameters are mainly representative of

<table>
<thead>
<tr>
<th>Sulfide</th>
<th>( \rho ) (ohm-m)</th>
<th>( m_0 ) (mV/V)</th>
<th>( \tau ) (s)</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden (OB)</td>
<td>250</td>
<td>10</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Disseminated sulfide (M)</td>
<td>500</td>
<td>350</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Conductor (S)</td>
<td>1</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Host rock (HR)</td>
<td>5000</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>
Figure 7. The LCI of the long-pulse 2015 VTEM synthetic quasi-2D sulfide deposit data without consideration of the IP effect. (a) True resistivity model. (b) True chargeability model. (c) Resistivity model recovered without modeling IP. (d) Data misfit.

Figure 8. Comparison of inversions of unprocessed versus processed predicted data over a synthetic quasi-2D sulfide model with starting Cole-Cole parameters: $\rho = 1000$, $m_0 = 100$, $\tau = 10^{-4}$, $\epsilon = 0.7$, and hard constraints for $\tau$ and $\epsilon$. (a) Inversion of unprocessed data (top, resistivity; middle, chargeability; and bottom, inversion misfit, normalized by standard deviation). (b) Inversion of processed data (top, resistivity; middle, chargeability; and bottom, inversion misfit, normalized by standard deviation). (c) Data fit, shown in transients (blue, noisy data; red, model) for unprocessed data. (d) Data fit, shown in transients (blue, noisy data; red, model) for processed data.
disseminated sulfides of relatively low concentration and small grain size. Examples of well-known deposits with similar properties, however, include Kidd Creek and Lornex deposits. Raiche (1998) plots the synthetic TEM responses expected over these deposits, showing IP effects clearly visible within the frequency bandwidth of the systems. The choice of the Cole-Cole values for this and all other models is based on published literature (from galvanic and EM sources), bound by the sensitivity limitations of the method. For example, a 25 Hz AEM system is insensitive to IP effects associated with \( \tau > 1 \text{ s} \) (Viezzoli et al., 2013). Macnae (2015) shows, under certain assumptions (step response in transmitter waveform, a target represented by thin sheet and \( c = 0.3 \)) that the range of maximum sensitivity to IP effects for such system is between \( 10^{-4} \) and \( 10^{-5} \) for \( \tau \) values.

For the true model, the IP effect metric (equation 1) was then calculated and the associated graph is shown in Figure 2c, with the noise floor. As shown in Figure 2c, the IP metric is at maximum over the chargeable disseminated sulfide body, located between stations 20 and 55. The metric, well above noise level, implies a locally measurable IP effect. This is confirmed by visual inspection of noise-contaminated transients (Figure 6d and 6e).

Results: Inversions of synthetic sulfide deposit data

Inversion without consideration of the IP effect

The first step was to invert the noise-contaminated synthetic data without consideration of the IP effect. These quasi-2D LCI inversion results are shown in Figure 7. As shown, inverting the IP-affected data without Cole-Cole modeling produces large data misfits above the chargeable body. In the model space (Figure 7c), the recovered resistivities are representative of the true model directly over the non-chargeable conductive body, whereas the recovered model is not well-recovered over the chargeable (disseminated) target.

Table 4. Statistical analysis of misfits derived from synthetic sulfide data inversions featuring scenarios considering unprocessed data, processed data, and locking parameter \( c \).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Inversions</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unprocessed</td>
<td>162</td>
<td>0.8221</td>
<td>313.1986</td>
<td>22.4091</td>
<td>64.1266</td>
</tr>
<tr>
<td>Processed</td>
<td>162</td>
<td>0.9203</td>
<td>36.7935</td>
<td>6.88230</td>
<td>10.9176</td>
</tr>
<tr>
<td>Locked ( c )</td>
<td>52</td>
<td>0.9470</td>
<td>63.8061</td>
<td>13.32853</td>
<td>20.6242</td>
</tr>
</tbody>
</table>

Figure 9. Unconstrained 1D inversion of processed full-waveform noisy synthetic VTEM data over the sulfide model with the starting Cole-Cole parameters: \( \rho = 1000, m_0 = 100, \tau = 10^{-4}, \) and \( c = 0.7 \). The red lines outline the true model.
Inversions in the IP mode

The next step was to invert with quasi-2D LCI, the noise-contaminated data using full multiparametric inversion (LCI) over 29 layers with simultaneous recovery of four Cole-Cole parameters. The results obtained using a starting model with a homogeneous half-space of \( \rho = 1000; m_0 = 100; \tau = 10^{-4} \); and \( c = 0.7 \) are shown in Figure 8a. The recovered parameters are masked to the DOI. The horizontal constraints were set to accommodate 10% covariance between neighboring parameters on \( m \) and \( \rho \), and “harder” constraints (0.1% covariance) on \( c \) and \( \tau \). The choice of constraints is due to expected limited variance of \( c \) and limited sensitivity of \( \tau \) to this parameter beyond the \( 10^{-4} \) to \( 10^{-2} \) range.

The resistive (disseminated sulfide) body is now well-resolved, with a vertical resolution in the line with expectations for this type of target/host rock combination. The chargeability section recovers a target extending to depth and centered on the true target. Some shallow strongly chargeable “ghost” artifacts are, however, also present in the inversion results. The latter can be attributed to the limited vertical sensitivity of the parameter \( m_0 \). In the next examples, we try to improve the results further experimenting with the quality of the inversion input (data processing), inversion regularization, and a priori information.

Inversions in the IP mode of processed data

Our experience with field AEM data (Viezzoli et al., 2012; Kaminski and Viezzoli, 2016) shows that assessing and eliminating noise before inversion improves its outcome in terms of robustness and accuracy of the recovered model parameters. We now attempt to prove this observation on synthetic data, in which the true model is known by introduction of the model norm metric, which provides quantitative insight to deviation of inversion results from the true model.

Although it is technically possible to use a constant noise floor for automated culling of background noise (even more so with synthetic data), our experience shows that the noise levels contaminating the recorded ground response can vary during a survey (Kaminski and Viezzoli, 2016). Therefore, any type of automated filtering requires manual refinement. The noise-contaminated data shown in Figure 6d have been first stacked laterally to increase S/N, then they are manually inspected and edited to eliminate noise at late times. The same inversion settings and starting models used in the previous section have been applied to invert the processed data. Figure 8b shows the inversion results, side by side with those obtained from unprocessed data (Figure 8a) in 2D sections. Processing the data to eliminate late time noise prior to inversion resulted in a slight, yet noticeable, improvement.
improvement in the recovery of the conductive and the chargeable targets. The latter, in particular, fills more uniformly, to its full extent, the outline of the disseminated sulfide. The near-surface artifacts are also reduced in the recovered chargeability section as a result of manual data processing.

**Testing different starting Cole-Cole parameters in multiparametric inversion**

The next step was to test the dependence of the simultaneous inversion of four Cole-Cole parameters on starting models selection and different regularization types. In general, the inverse problem is underdetermined and considering four varying parameters, the problem can become unstable and sensitive to starting models.

Inverting with consideration of the full suite of different starting models combinations (Table 2) yields a total of 81 combinations. All of these combinations were tested with two types of regularization on $\tau$ and $c$ parameters: soft constraints (allowing 10% variance between neighboring horizontal and vertical locations) and hard constraints (allowing 0.1% variance between neighboring horizontal and vertical locations). A total of 162 inversions were therefore carried out first on the unprocessed, noise-contaminated data shown in Figure 6d, then on processed data. The inversion results were then assessed by misfit values. Global misfit values, normalized by standard deviations of unprocessed data inversions, range from 0.82 to 313.20

depending on the starting parameters and type of constraints. The processed data inversion results were subject to similar misfit analysis and comparison of misfits with those achieved for unprocessed data inversions (Table 4).

The misfits normalized by standard deviation for processed data range from 0.92 to 36.79. Their average and standard deviation values are significantly lower than for the unprocessed data. There are, nonetheless, high misfits still associated with a significant number of inversion results. This is due to combinations of unrealistic starting models, which prevented convergence. Furthermore, the lowest data misfit achieved for processed data inversion is actually slightly higher than the lowest misfit for unprocessed data inversion, which is a result of eliminating noisy late times during processing, associated with large data misfits and, therefore, easier convergence.

We have shown that processing data generally reduces misfits, and it assists in recovery of better models. The nonuniqueness, however, still remains significant even for processed data, hence it is still essential to test a range of starting parameters for Cole-Cole inversions. As with any other ill-posed problem, the use of ancillary information in the form of a priori information (drilling, downhole, ground geophysics, etc.) reduces the ambiguity of the results and allows narrowing down the number of plausible recovered models. The visual inspection of the 162 inversion models proves, for example, that the use of tight spatial constraints on the $c$ and $\tau$ parameters often improves the match between recovered and true models.

Figure 11. The LCI of processed full-waveform noisy synthetic VTEM data over quasi-2D sulfide model with starting Cole-Cole parameters: $\rho = 1000$, $m_0 = 100$, $\tau = 10^{-4}$, $c = 0.7$, and hard constraints on $\tau$ and $c$ and locked resistivities and chargeabilities for stations 50 and 60 (profile distances 1470 and 1770 m, respectively).
without a significant increase in misfit. The latter confirms that sensitivity to $c$ and $\tau$ remains limited; however, these parameters should be still inverted for, not locked (as shown in the next section).

Testing different types of constraints and a priori information on the synthetic sulfide deposit model

In the following exercises, the inversions were carried out for processed data, testing different combinations of lateral constraints and a priori information inputs. They illustrate the improvement brought by adding extra data that reduce the ill posedness of the problem.

The synthetic data were inverted in entirely 1D mode, i.e., without any lateral or spatial constraints and as a sequence of individually inverted soundings. The results are shown in Figure 9. The comparison with Figure 8b provides a clear visual assessment of the effects of the lateral constraints on resistivity and chargeability. Although the data misfit is similar, the Cole-Cole parameters recovered in Figure 9 display a larger degree of variance. Due to

Figure 12. The LCI of processed full-waveform noisy synthetic VTEM data over quasi-2D sulfide model with starting Cole-Cole parameters: $\rho = 1000$, $m_0 = 100$, $\tau = 10^{-4}$, $c = 0.3$, and hard constraints on $\tau$ and locked $c$.

Figure 13. Mode calculated for the logarithmic differences between the true and recovered chargeability models, which have produced misfits less than 2 (normalized by standard deviation). (a) Inverting for $c$. (b) With locked parameter $c$. 
the absence of lateral constraints, such variance is, in many places, exaggerated with respect to the true model.

Another advantage of applying the LCI/SCI lies in its capacity of spreading robustly the localized a priori information that might originate from, e.g., boreholes. In the next examples, a priori information was applied, associated with lateral constraints of varying strengths. The a priori information enters the inversion as another data set, with its own uncertainty. The LCI seeks a result that balances the information from AEM data, a priori, and lateral constrains. The a priori was introduced for stations 50 (profile coordinate: 1470) and

Table 5. Cole-Cole parameters for synthetic kimberlite model.

<table>
<thead>
<tr>
<th>Kimberlite</th>
<th>( \rho ) (ohm-m)</th>
<th>( m_0 ) (mV/V)</th>
<th>( \tau ) (s)</th>
<th>( c ) (N/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden (OB)</td>
<td>500</td>
<td>10</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Crater facies (S)</td>
<td>30</td>
<td>300</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Diatreme facies (M)</td>
<td>250</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Host rock (HR)</td>
<td>5000</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Figure 14. Synthetic quasi-2D kimberlite true model. (a) Resistivity model (top 150 m). (b) Chargeability model (top 150 m). (c) IP effect metric (blue curve), compared with system noise (red line).

Figure 15. The LCI of the synthetic kimberlite VTEM data without consideration of the IP effect. (a) True resistivity model. (b) True chargeability model. (c) Resistivity model recovered without modeling of the IP effect. (d) Associated data misfit.
60 (profile coordinate: 1770), locking the resistivity and the charge-ability parameters at their true values, as if they had been gathered from direct measurements. Then, hard lateral constraints were introduced, increasing the spatial coherence of neighboring stations. The results in Figure 10 show an improvement in the geometry of the recovered targets, although they are rather localized around the stations with available a priori. Tightening the lateral constraints further extrapolates the effect of a priori farther away from their source (Figure 11). As a result, the disseminated sulfide model is better recovered in terms of its resistivity and chargeability values. The geometry of the target is also closer to true model.

The recovered \( c \) and \( \tau \) models require additional discussion. Their true values were set to half-space (Table 3). Figure 10 shows the results obtained solving for \( c \) and \( \tau \), but without lateral constraints. In places, the recovered \( c \) value has a large, localized artificial anomaly (e.g., around profile coordinate 1300). This is associated with heavily suppressed chargeability. The presence of lateral constraints stabilizes the inversion and reduces significantly such artifacts. With application of lateral constraints, the recovered relaxation time constant \( \tau \) remains close to the starting value, with the output models changing almost insignificantly from the starting models. The only exception is perhaps shown in Figure 11, which displays a slight increase in \( \tau \) in correspondence to the disseminated sulfide model. The frequency parameter \( c \), on the other hand, generally shows more structure with the output values that differ significantly from the starting models.

The latter can be attributed to the very significant effect that changing \( c \) has on the slope of the transients (Walker and Kawasaki, 1988; Viezzoli et al., 2013). As a consequence, we anticipate that locking the frequency parameter to a predefined constant value can result in high misfits and/or artifacts in some of the other Cole-Cole models recovered.

To further illustrate the effect of locking \( c \) parameter, we have inverted the synthetic data set with constant \( c = 0.3 \) (half-space) and not allowing it to vary. All other inversion parameters were kept the same as in the case shown in Figure 8.

Figure 12 shows the inversion results with locked parameter \( c \). With the use of certain starting model parameters, it became numerically possible to achieve satisfactory data misfit in the data space. However, in model space, the differences between locking \( c \) and inverting for \( c \) are obvious. For example, in the chargeability section (\( m_0 \), Figure 12), the chargeable material distribution is skewed toward the near surface and at the same time is recovered, suggesting larger absolute values. When Figure 12 is compared with Figure 8, the match with the true model is visibly better for the model shown in Figure 8. In the case when \( c \) is locked, the \( m_0 \) distribution is skewed. Numerically, this skewness is reflecting compensation for the impossibility of recovering structure in the \( c \) parameter domain. The recovered relaxation time \( \tau \) appears to be recovered below its true value. The resistivity model is not significantly affected.

Figure 16. The LCI of synthetic quasi-2D kimberlite VTEM data with starting Cole-Cole parameters: \( \rho = 1000, m_0 = 50, \tau = 10^{-3}, c = 0.3 \), and soft constraints on \( \tau \) and \( c \).
We also ran the full suite of inversions, scanning the entire range of starting models (Table 2) with the exception of $c$, which remained locked at 0.3 value and have further analyzed the results in terms of misfits and divergence from true model (Table 4). The misfit is generally increased, and it displays a higher variance with respect to solving for $c$.

To analyze the statistics in model space, we calculated the mode of the differences in chargeability between the true and output models, described in the “Methodology” section. The population was limited to the chargeability models associated with misfits less than 2. The results (calculated in log space) are shown in Figure 13a and 13b. The values around zero are representative of the smaller difference between the true and the recovered models. The comparison is only meaningful in the proximity of the synthetic disseminated sulfide, in which the data are expected to have some sensitivity to $m_0$. Figure 13 shows that inverting for the $c$ parameter, rather than locking it, yields lower values of the mode of the differences. The output models obtained from inversions with varying $c$ parameter are therefore statistically more often closer to the true models.

**SYNTHETIC KIMBERLITE MODEL**

**True model**

The true model of the kimberlite is shown in Figure 14a and 14b. It consists of four general rock types: overburden (OB), crater facies of kimberlite (S), diatreme facies of kimberlite (M), and the host rock (HR). The true model parameters are provided in Table 5.

**IP effect metric and noise contamination**

The model shown in Figure 14a and 14b was used to forward simulate the responses of the same VTEM system used previously. Similar modeling considerations were in effect for the synthetic kimberlite model, as in the case with the synthetic sulfide model. The synthetic VTEM data set, produced with Cole-Cole forward modeling for the IP effect, was then contaminated with noise.

Then, the IP effect metric (equation 1) was calculated (Figure 14c). The latter suggests that the IP effect should be detectable above the noise.

**Results: Inversions of synthetic data**

**Inversion without consideration of the IP effect**

The first step was to carry out the quasi-2D LCI on the synthetic data without consideration of the IP effect. The results are presented in Figure 15. It is clear that inverting the IP-affected data without the consideration of the IP effect produces a large (7.4) misfit in the data space. In the model space, the inversion recovers a reasonable resistivity section, which, however, does not resolve the...
overburden and slightly underestimates the depth to the crater facies of the kimberlite. No information on the chargeability distribution is obtained.

**Inversions in the IP mode**

The results of the quasi-2D LCI (29 layers) in the IP mode for the kimberlite model are presented in Figure 16. They have been obtained with a starting model with a homogeneous half-space of $\rho = 1000$, $m_0 = 50$, $\tau = 10^{-3}$, $r = 0.3$, and soft constraints on $\tau$ and $c$. The resistivity section is slightly closer to the true model, especially the crater facies (the red line shows the contours of the true model). The chargeability section provides the satisfactory recovery of the geometry of the crater facies. The displayed lateral variability of parameter $m_0$ in the model can be attributed to the presence of noise in the data, as well as to the fact that the sensitivity to $m_0$ distributions at those depths is moderate under implemented soft constraints. As per the general LCI approach (Christiansen and Auken, 2012), tightening up the constraints would increase the lateral coherence of the parameters. The shallow artifacts visible in the overburden are mainly associated with the reduced sensitivity to the thinning target at the edges of the crater facies. The rather homogeneous $c$ and $\tau$ sections are indicative of the limited sensitivity to both parameters.

**Testing different types of constraints and a priori information**

A priori information was introduced for station 55, locking resistivity and chargeability parameters at their true values and expanding their effect to neighboring points by assigning tighter lateral constraints on these parameters. This slightly increased the accuracy of the recovered crater facies’ geometry (Figure 17), in the resistivity and chargeability sections. The near-surface chargeable artifacts in the overburden are also reduced. The $c$ section in this case shows some structure corresponding to the crater facies. This is due to the extra information provided as input to the inversion (the a priori from boreholes and the tighter constraints).

<table>
<thead>
<tr>
<th>Simple deep model</th>
<th>$\rho$ (ohm-m)</th>
<th>$m_0$ (mV/V)</th>
<th>$\tau$ (s)</th>
<th>$c$ (N/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Host rock #1</td>
<td>1000</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Host rock #2</td>
<td>50</td>
<td>1</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Host rock #3</td>
<td>1000</td>
<td>1</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Target</td>
<td>30</td>
<td>300</td>
<td>0.001</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 18. The LCI of 2015 VTEM full-waveform synthetic data with starting Cole-Cole parameters: $\rho = 1000$, $m_0 = 50$, $\tau = 10^{-3}$, $c = 0.3$, and soft constraints on $\tau$ and $c$, a priori information and released vertical constraints were used for this inversion.
Finally, we ran another inversion with the same starting model and a priori information of the previous example, but releasing completely the vertical constraints on \( m_0 \) and \( \rho \) (Figure 18). This test emphasized the horizontal interfaces, producing more blocky results in the vertical direction and more smooth results in the horizontal direction. As a consequence, the \( c \) section is also recovered with greater accuracy.

**“DEEP” MODELS**

Two more synthetic models have been designed to study the detectability and behavior of the IP effect arising from deeply buried targets, as well as in the absence of the IP effect. These models, which do not aspire to represent any specific type of geology or target, include

- the “dipping chargeable layer” model, in which the depth to conductive and chargeable layer increases along the profile
- The “buried conductor,” a modified version of the previous model with a conductive, nonchargeable basement placed below the dipping conductive and chargeable layer.

**Dipping chargeable layer model (“simple deep” model)**

In this exercise, series of conductive and chargeable targets are placed at increasing depth along the profile (Figure 4), resembling a dipping layer. The surficial rock resistivity is split in half, resembling a conductive (right) and a resistive (left) overburden, both

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![Figure 19. The LCI of quasi-2D simple deep simulated data in the no-IP mode.](#)

![Figure 20. The LCI of synthetic VTEM data for a quasi-2D simple deep model in the IP mode with the following starting model parameters: \( \rho = 1000, m_0 = 10, \tau = 10^{-3}, c = 0.5 \), and hard constraints on \( \tau \) and \( c \).](#)
nonchargeable. Table 6 contains the Cole-Cole parameters used for this model, chosen to have high sensitivity to the chargeable target ($\tau = 1$ ms).

The insert in Figure 4 shows the forward-modeled transients calculated over the left half of the model. As expected, the trough associated with the IP effect migrates to later times as the layer dips toward the center of the model.

**Inversions of synthetic data**

**Inversion without consideration of the IP effect**

These simulated VTEM data were first inverted using the 2D-LCI approach in the “no-IP mode” using $\rho = 1000$ for the starting model of the homogeneous half-space (Figure 19). The recovered resistivity section is producing high misfit in the data space. Below the resistive overburden, the depth to the conductor is underestimated and its thickness is overestimated. The inaccuracies of the recovered model increase in the presence of a conductive host rock (right part of the cross section).

**Inversions in the IP mode**

The data were further inverted with a quasi-2D LCI inversion in the IP mode (Cole-Cole modeling), using a subset of the full suite of starting model combinations described in Table 2. Figure 20 shows the results obtained using the starting model parameters: $\rho = 1000$,

<table>
<thead>
<tr>
<th>Sulfide</th>
<th>$\rho$ (ohm-m)</th>
<th>$m_0$ (mV/V)</th>
<th>$\tau$ (s)</th>
<th>$c$ (N/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden</td>
<td>1000</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Host rock</td>
<td>1000</td>
<td>1</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Shallow conductors</td>
<td>30</td>
<td>300</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Conductor #1</td>
<td>10</td>
<td>1</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>Conductor #2</td>
<td>1</td>
<td>1</td>
<td>0.001</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 21. Sensitivity to chargeability calculated for quasi-2D simple deep model.

Figure 22. Synthetic quasi-2D deep-conductor true model (fixed $\tau$ and $c$ parameters to $10^{-3}$ and 0.5, respectively); the top 400 m are shown.

Table 7. Cole-Cole parameters for deep conductor synthetic model.

m_0 = 10, \tau = 10^{-3}, c = 0.5, \text{ and hard constraints on } \tau \text{ and } c. \text{ As discussed earlier, the hard constraints reduce the ill posedness of the inversion problem and the ambiguity of the results. The chargeable and conductive dipping layer is recovered rather accurately to depths in excess of 100 m under resistive host rock and to depths not exceeding 50 m under conductive host rock. These observations are in agreement with the sensitivity analysis (Auken and Christiansen, 2004) for the true chargeability model as shown in Figure 21. The 2D section clearly shows the area of higher sensitivity associated to the dipping layer. The sensitivity is reduced by the presence of the conductive overburden to the right. The presence of conductive host rock also reduces the accuracy of the recovered resistivity section, although it remains in better agreement with the true model than the section obtained without IP modeling.}

“Buried conductor” model

The purpose of this experiment was to illustrate the potential masking effect of a shallower conductive and chargeable layer on a buried conductive basement. We have used a slightly modified version of the model described in the previous section. A buried conductive basement with changing conductivity was added at depth of 260 m. The

Figure 23. The LCI of IP-affected data in no-IP mode (third panel — resistivity only) with a 1000 ohm-m starting model. The red lines outline the reference model.

Figure 24. (Top) Reference quasi-2D model (no-IP). (Bottom) Laterally constrained resistivity inversion in the no-IP mode with 1000 ohm-m starting model and plotted misfit. The red lines outline the reference model.
true model is shown in Figure 22; the Cole-Cole parameters of the true model are shown in Table 7. The model is similar to the one used in a previous experiment (Figure 20), which has produced a measurable IP effect, and it is therefore suitable for testing if IP effects can mask buried conductors.

Inversion without consideration of the IP effect

As in previous examples, the IP-affected, noise-perturbed synthetic VTEM data were inverted for resistivity only, without consideration of the IP effect (Figure 23). The data misfit is high. The buried conductors are not recovered in the inversion. It is most likely that this is due to the presence of a shallower chargeable and conductive layer. At this stage, it is not clear whether it is the chargeable or the conductive characteristics (or their combination) of this shallower layer that are responsible for masking the buried conductor. To sort this, we ran another experiment, using a new model, identical to the previous, but for the total absence of chargeable material (Figure 24). The IP-free VTEM data set was inverted (in the no-IP mode), using a starting model with homogeneous half-space of 1000 ohm-m. The inversion results are presented in Figure 24 (lower panel). The basement conductors are now recovered, even though the more conductive basement to the right is rendered deeper than the true model. This is due to the vertical constraints applied on the resistivity, which smear the sharp transition between host rock (1000 ohm-m) and basement (1 ohm-m).

It is conclusive that it is the chargeability in the dipping shallower layer that hindered the recovery of the conductive basement in the previous experiment. It is, however, possible to adequately image the deep-seated conductor with proper differentiation of its varying electrical conductivity.

The final question is whether, in the presence of the shallow dipping chargeable layer, the basement conductor can be recovered if it is inverted in the IP mode, or whether it will remain unresolved. This is assessed in our next numerical experiment.

Inversions in the IP mode

The IP affected, noise added synthetic VTEM data associated with the model of Figure 22 and Table 7 were inverted in the IP mode (Cole-Cole modeling), using a subset of the full suite of previously described Cole-Cole parameter combinations. Figure 25 shows the results obtained with following starting half-space parameters: \( \rho = 100, \ m_0 = 100, \ \tau = 10^{-3}, \ c = 0.3 \), and soft constraints on \( \tau \) and \( c \). The data misfit is homogeneously low along the entire section. The resistivity of the dipping layer is well-resolved, whereas the chargeability is resolved to a fair degree. The deep conductors are recovered fairly well in the middle of the section, even though the vertical constraints create an artificial offset at the boundary between the two halves of the conductive basement. As discussed earlier, the constraints seem to smear the otherwise very sharp boundary between the host rock and the 1 ohm-m basement. The conductive basement is less well-recovered toward the edges of the section, where the IP effect is the strongest, due to the shallower burial depth of the chargeable layer. This means that, by virtue of IP modeling, one may or may not allow resolving otherwise masked bedrock conductors, depending on the magnitude of the IP effect associated with the shallower chargeable layers.

CONCLUSION

This study reports and analyzes the results of numerical experiments carried out with quasi-2D forward of AEM data and subsequent inversion of a series of models with chargeability distributions. We describe an approach to multiparametric quasi-2D inversion of AEM data affected by IP, using Cole-Cole modeling and lateral constraints. The conclusions derived herein are based on the range of Cole-Cole parameters used in our experiments (e.g., \( \tau \) range between \( 10^{-2} \) and \( 10^{-4} \) s).

This ill-posed problem calls for careful use of ancillary information in the inversion, in the form of both lateral constraints and, when possible, of a priori information. The range of the starting model parameters should be properly sampled, within the expected sensitivity range of the AEM system because having the starting
model values too far from the true values can prevent convergence and/or produce artifacts in the output models. As expected, the data showed limited sensitivity to $r$ and $c$ parameters. It is nonetheless important to invert for these parameters, possibly using hard constraints, rather than locking them, to avoid near-surface skewness and other artifacts in the chargeability distribution. Prior to the inversions, it is also important to perform careful preinversional processing of the AEM data to assess and reduce the effects of noise. The output models should always be assessed against any available ancillary information to reduce ambiguity.

Our results prove that this approach can be very useful in cases in which standard EM modeling (conductivity-only) yields inaccurate results, due to the presence of IP effects. Examples of improvements in EM modeling using our multiparametric approach include:

- improved resistivity cross sections (IP effects can cause inaccurate resistivity estimates if not modeled);
- detection of chargeable targets (also in the absence of significant resistivity contrast), with an associated estimate of the DOI;
- detection of buried, dipping chargeable, and conductive layers to depths in excess of 100 m;
- detection of basement conductors buried below chargeable layers that, in some cases, can mask them if the IP effect is not modeled;
- improved interpretation.

All these described advantages contribute to a potential significant improvement in interpretation for geologic mapping and mineral exploration. When dealing with real AEM data sets, usually acquired with parallel lines, this approach should be slightly modified to quasi-3D, using the SCI. The spatial constraints are an effective tool to further stabilize the inversion.

Several questions remain open and are subject to ongoing research. Examples include automated data processing customized for IP effects, or greater flexibility in the inversion scheme, which better reflects the sensitivity of the different model parameters (e.g., start inverting only for $\rho$ and $m_0$ during the first few iterations, then introducing $c$, then $r$). Models different from Cole-Cole can also be tested. A similar systematic numerical analysis could also be carried out for B-field data, which, bringing the sign change to earlier times, are arguably better suited for extracting IP information from AEM. The level of inaccuracy of applying the 1D approximation, and its significance to real field data from large-scale surveys remains to be assessed. Notwithstanding all these potential improvements, the methodology presented herein is robust and appropriate to produce significant impact on real AEM data sets over different targets and with different AEM systems from around the world.

REFERENCES


Walker, G. G., and K. Kawasaki, 1988, Observation of double sign reversals during the first few iterations, then inverting only for $\tau$ and $m_0$ during the first few iterations, then introducing $c$, then $\tau$). Models different from Cole-Cole can also be tested. A similar systematic numerical analysis could also be carried out for B-field data, which, bringing the sign change to earlier times, are arguably better suited for extracting IP information from AEM. The level of inaccuracy of applying the 1D approximation, and its significance to real field data from large-scale surveys remains to be assessed. Notwithstanding all these potential improvements, the methodology presented herein is robust and appropriate to produce significant impact on real AEM data sets over different targets and with different AEM systems from around the world.