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A novel approach to comparing AEM inversion results with borehole conductivity logs

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Abstract. Borehole conductivity logs, besides being useful for identifying, interpreting and correlating geological formations, also find widespread use as auxiliary information in the inversion of airborne electromagnetic (AEM) data. One of the quality checks often applied to AEM inversion results is a comparison between the conductivity structures revealed by borehole conductivity logs in the survey area and the AEM inversion model closest to the borehole, often called an 'FID point comparison'.

Another use of borehole conductivity logs is found in modern AEM inversion procedures, where the borehole conductivity information is included as prior information in a laterally constrained inversion. In most former and present practices, AEM layer conductivities are compared with the measured conductivity in the borehole. However, the borehole conductivity is essentially an apparent conductivity – it is a measured data value – while the AEM layer conductivities are model parameters resulting from inverting AEM data. To avoid comparing data and model parameters we suggest a conceptually clear approach based on an inversion of the borehole conductivity *data* to obtain a borehole conductivity *model*, which in turn can be compared with the AEM model. Furthermore, the AEM forward response of the borehole model can, in a consistent way, be compared with the AEM data. In both approaches, we keep track of uncertainty and define quantitative, uncertainty-normalised measures of the difference between borehole and AEM values, and we find simple functional relationships between the two. The methodology is demonstrated on the AEM data and conductivity logs of the Broken Hill Managed Aquifer Recharge (BHMAR) project.

Key words: airborne, borehole constrained inversion, borehole log, comparison, electromagnetic.

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Introduction

Traditionally, the correspondence between borehole conductivity logs and airborne electromagnetic (AEM) inversion models has been expressed through the so-called 'FID point comparison (comparing borehole log results with the AEM model with the FIDucial, i.e. the location, closest to the borehole)'. In this approach, the borehole conductivity log is compared with the model conductivities obtained from inversion of AEM data from the location closest to the borehole (Lawrie et al., 2000, 2009, 2010, 2012*a*; Lane et al., 2001; Schamper et al., 2014). In this paper, we consider AEM methods, but our methodology and arguments are equally valid for comparisons between borehole conductivity logs and any electrical or electromagnetic (EM) method.

Increasingly, there is a demand from policy and other stakeholders funding groundwater investigations for improved parameterisation of numerical groundwater models as well as more reliable quantitative assessments of the uncertainties and confidence levels in model predictions. Recent advances in the ability to map and characterise key elements of hydrogeological systems using AEM methods and the integration of AEM data with borehole geological and geophysical data to provide maps of derived hydraulic parameters (Foged et al., 2014; Lawrie et al., 2009, 2010, 2012*a*), has seen a more frequent use of AEM data as key inputs into groundwater models (Munday

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et al., 2003; Walker et al., 2004; KBR, 2010, 2011) and groundwater management plans (Strategen, 2014).

With this increased up-take of AEM data has come heightened scrutiny of AEM inversion models and requests from stakeholders, including the multi-disciplinary groundwater science community, for optimised inversion models that map the subsurface conductivity with a high degree of confidence (Lawrie et al., 2012a, 2015), while also providing transparency in estimates of inversion model uncertainty (Lawrie et al., 2012b). In Australia, the use of borehole conductivities in constrained inversion approaches is recognised as 'best practice' in fitting the knowledge of the subsurface conductivity from borehole logs with more remote AEM data (Spies and Woodgate, 2005). The comparison of borehole conductivity data (not used in the constrained inversions) with the AEM response is the traditional way of assessing the agreement between the borehole logs and the inversion models (Lawrie et al., 2000; Lane et al., 2001).

There are several issues that need to be acknowledged in connection with assessing possible inconsistencies between borehole conductivity logs and the models resulting from AEM inversion. The basic problem with the comparison is the huge scale difference between the volume occupied by the sensitivity function of the log tool and the corresponding volume for the sensitivity function of an AEM measurement. Independent of depth and conductivity structure, the sensitivity of a log tool with 0.50 m separation between the induction coils will lie within a vertical distance of ~1 m, centred on the midpoint between the coils, with a radius of ~1 m, a volume of the order of a few cubic metres. The volume occupied by the AEM sensitivity function is mainly controlled by the diffusion length, which is proportional to $\sqrt{t/\sigma}$, where *t* is the delay time and σ is the average conductivity of the ground. At early times, in a conductive earth, the depth extent of the sensitivity function for most AEM systems would be at least 5 m and the radial extent at least 50 m (Christensen, 2014), so the ratio between the log tool and AEM sensitivity volumes is of the order of 1 : 10⁴. For late times, the ratio can easily be smaller than 1 : 10⁷. Furthermore, while the vertical resolution of the log tool is the same at all depths, the vertical resolution capability of the AEM system decreases with depth.

The scale issue is a long and complicated discussion worthy of its own research, and we have not gone into details here. However, it must be kept in mind that in a comparison between conductivity values coming from geophysical methods with such disparate sensitivity distributions, there will potentially be many cases of misleading and irrelevant results. In essence, almost any log result can be achieved by anomalies around the borehole that are so small relative to the sensitivity volume of the AEM system that they have very little influence on the AEM measurements and the resulting AEM inversion model. Such small anomalies can be quite irrelevant for the (hydro)geological interpretation. This situation means that both relevant and irrelevant matches/ mismatches can arise in the comparison, and there is no way to discern whether the results of an individual comparison is relevant or not.

Geologically, the scale issue is related to small-scale variability in electrical conductivities driven by heterogeneities in lithologies (texture, mineralogy and porosity) and groundwater salinities (Lawrie et al., 2000, 2009, 2010, 2012b). Close-spaced drilling in the Ord Valley (Lawrie et al., 2010) and the Broken Hill Managed Aquifer Recharge (BHMAR) (Lawrie et al., 2012a, 2012b) projects revealed significant lateral and vertical heterogeneity in both siliciclastic and carbonate sedimentary systems within the scale of the AEM footprint. These variations can be considerable and can contribute to poorer correlations between borehole and AEM conductivity data (Lawrie et al., 2010, 2012a). Such local-scale heterogeneity can be anticipated in some sedimentary systems (e.g. some braided alluvial channels), while additional effects related to overprinting weathering/diagenetic/structural effects on porosity distribution, mineralogy, and consequently, electrical conductivity, can also be significant at local scales (Lawrie et al., 2000, 2010, 2012a). Small-scale heterogeneity in electrical conductivity driven by variability in pore-fluid salinities is also common in some near-surface environments, notably in highly salinised Australian landscapes (Lawrie et al., 2009, 2010, 2012a).

In the light of the scale issue, it seems unfounded when the comparison is expressed as one of 'verifying the AEM results by comparing with the (implied: true) borehole conductivity' as has been frequently done in the past. The methodology demonstrated in this paper stresses that a 'perfect' correlation between borehole and airborne conductivity measurements should not be anticipated in most geological settings, due in large to the local variability in lithologies and pore-fluid compositions, as described above. Fortunately, there is an increasing awareness of this, also expressed in recent publications (Ley-Cooper and Davis, 2010). In this paper, we have deliberately avoided the phrase 'confirm' in relation to FID point comparison methods.

Another issue that warrants some remarks is the fact that the boreholes made after an AEM investigation are not randomly distributed; their locations are heavily biased in areas that carry the most potential (groundwater, mining opportunities, etc.) for the project in question, judging from the AEM results. The whole purpose of an AEM dataset is actually to provide the basis for choosing borehole locations for further investigations in an optimal fashion, i.e. so that the likelihood of hitting a target is optimised within the financial limitations of the drilling program. Borehole locations can be biased towards the more homogeneous parts of a survey area or the more inhomogeneous parts, and this of course gives bias to the interpretation of the FID point comparison results as an indicator for the whole survey area.

The scale problem and the biased location of the boreholes are inherent to any comparison between borehole conductivity logs and conductivity information from other sources, and naturally, the novel FID point comparison methodologies presented in this paper do not in any way (re)solve this issue. The issue that we do try to solve with the methodology in this paper is the fact that, in traditional FID point comparison, AEM models are compared with (averages of) conductivity log data. The AEM models are found through inversion, most often by assuming a 1D conductivity distribution, while the conductivity logs are data indicating an apparent conductivity. A conceptually more clear approach to the comparison would be to avoid comparing numbers from model space and data space, and instead make a comparison in the model space for both methods, referred to as the ModMod comparison, or in the data space as the DatDat comparison.

The ModMod comparison can be achieved by inverting the conductivity log data with the same multi-layer 1D model as the one used in the AEM inversion and then performing the comparison between model layer conductivities. The DatDat comparison can be carried out by first inverting the conductivity log data to obtain a 1D model and then forward calculating the response of that model for the AEM system and comparing it with the actually measured data at the FID point position.

An important aspect of the DatDat comparison is that it does not depend on the actual AEM inversion program used. There are very many different AEM inversion programs, but the DatDat comparison is independent of the inversion program as long as the program is capable of producing correct responses and derivatives, i.e. to model the forward responses correctly, taking the full system response of the AEM system into account (Christiansen et al., 2011). The DatDat comparison thus offers a unique opportunity of assessing the possible inconsistency between borehole logs and the AEM data independently of the differences in AEM inversion procedures. In this respect, it resembles the methodology of the calibration approach for EM systems in Foged et al. (2013) where comparisons are done exclusively in the data domain based on a generally accepted 'true' model for the calibration site.

We develop the two self-consistent approaches of comparison, the ModMod and DatDat comparisons, taking uncertainty from both methods into account. First, we present a short introduction to the inversion approach used in the AEM and log inversions. We then present ModMod and DatDat comparisons for the field example of the BHMAR project.

Theoretical framework

Inversion methodology

There are numerous approaches to the inversion of electrical and EM data with a 1D model consisting of horizontal, homogeneous and isotropic layers. The model used in the inversions of this paper is a multi-layer model, sometimes called a smooth model, where the subsurface is divided into a large number of layers. In the iterative inversion, the layer boundaries are kept fixed and only the log(conductivities) of the layers are changed in the inversion. The inversion formulation used in this paper is a well-established iterative constrained least-squares approach (Menke, 1989). Formally, the model update at the *n*'th iteration is given by:

$$\boldsymbol{m}_{n+1} = \boldsymbol{m}_n + \left[\boldsymbol{G}_n^T \boldsymbol{C}_{obs}^{-1} \boldsymbol{G}_n + \frac{1}{\sigma_v^2} \boldsymbol{C}_m^{-1}\right]^{-1} \cdot \boldsymbol{G}_n^T \boldsymbol{C}_{obs}^{-1} (\boldsymbol{d}_{obs} - \boldsymbol{g}(\boldsymbol{m}_n))$$
(1)

where m is the model vector containing the log(conductivities), G is the Jacobian matrix containing the derivatives of the data with respect to the log(conductivities), T is the matrix transpose, C_{obs} is the data error covariance matrix, C_m is a model covariance matrix imposing a vertical smoothness constraint on the multi-layer model, d_{obs} is the field data vector and $g(m_n)$ is the non-linear forward response vector of the *n*'th model. In this study, as is most often the case, the data noise is assumed to be uncorrelated, implying that C_{obs} is a diagonal matrix. σ_v is the standard deviation of the covariance matrix controlling the strength of the vertical constraints.

The model parameter uncertainty estimate relies on a linear approximation to the posterior covariance matrix, C_{est} , given by:

$$\boldsymbol{C}_{est} = \left[\boldsymbol{G}^{T}\boldsymbol{C}_{obs}^{-1}\boldsymbol{G} + \frac{1}{\sigma_{v}^{2}}\boldsymbol{C}_{m}^{-1}\right]^{-1}$$
(2)

where G is based on the model achieved after the last iteration. The analysis is expressed through the standard deviations of the model parameters obtained as the square root of the diagonal elements of C_{est} (e.g. Inman et al., 1973).

In this study, a 30-layer model is used where the depths to the layer boundaries increase downwards as a hyperbolic sine of the layer number. In this way, the depths to the layer boundaries increase linearly for small depths so that the top layers are approximately all of the same thickness, and the depths to the layer boundaries increase exponentially at large depths so that the thickness of a layer is a factor times the previous one. The thickness of the top layer is 0.5 m and the depth to the lowermost layer boundary is 200 m.

Methods of calculating the transient response from AEM systems can be found in Ward and Hohmann (1987). The inversion strategy used for the AEM data is described in Christensen et al. (2009) and Christensen (2016*a*, 2016*b*), and further details will not be provided here.

The model covariance matrix

We shall adopt a model covariance matrix based on a von Karman covariance function. The general expression for these functions is:

$$\Phi_{\nu,L}(z) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{|z|}{L}\right)^{\nu} K_{\nu}\left(\frac{|z|}{L}\right)$$
(3)

where z is the depth, K_v is the modified Bessel function of the second kind and order v, Γ is the gamma function, L is the maximum correlation length accounted for and σ controls the amplitude. For $v \rightarrow 0$, the von Karman function effectively contains all correlation lengths due to the logarithmic singularity of K_0 . This broadband behaviour ensures superior robustness in the inversion, i.e. model structure on all scales will be permitted if required by the data, and it makes the regularisation imposed

by the model covariance matrix insensitive to the discretisation (Serban and Jacobsen, 2001; Christensen et al., 2009).

A good approximation to the von Karman functions that allows rapid calculation and analytical integration over model elements can be achieved by stacking single-scale exponential covariance functions with different correlation lengths (Serban and Jacobsen, 2001):

$$\Phi_{\nu,L_N} \approx \sigma_{\nu}^2 \sum_{n=0}^{N} C^{n \cdot \nu} \exp\left(-\frac{|z|}{C^n L_N \cdot 0.65}\right)$$
(4)

where L_N is the maximum correlation length represented, *C* is the factor (*C* < 1) between the correlation lengths, *N* is the number of stacked single-scale covariance functions and σ_v is the standard deviation of the correlation. The factor 0.65 in the exponential denominator is an empirical factor that ensures the fit to the von Karman function. The resulting stacked covariance function is essentially free of correlation scale. The lower and upper limits of the correlation lengths are a mathematical convenience and do not influence the correlation properties at the distance scales typically studied.

In this study, the parameters v=0.1, C=0.1, $L_N=10000$ km and N=9 have been used. This means that the covariance function will contain correlation lengths between 6500 km and 6.5 cm, one per decade. This covers scales of geological variability between the radius of the Earth and small stones, clearly sufficient for the resolution capability of airborne transient electromagnetic (TEM) data. Notice that the model covariance matrix only depends on the geometry of the multi-layer model, so it needs to be calculated and inverted only once.

Inversion of borehole conductivity log data

For the comparison between AEM and borehole information, the borehole conductivity log data have been inverted with the same multi-layer model as the AEM data. This has been done to establish complete equivalence between the way the AEM data and the borehole data contribute to the overall information about the subsurface conductivity.

In the low-frequency approximation, the induction log response and the apparent conductivity is a linear function of the formation conductivities:

$$\sigma_a(z) = \int_0^\infty S(z - z') \,\sigma(z') \,dz' \tag{5}$$

where S(z) is the sensitivity function for the conductivity log and $\sigma(z)$ is the conductivity as a function of depth.

In the low-frequency approximation, for a two-coil system with a coil distance of L, S is model independent and is given by:

$$S(z) = 1/(2L)$$
 for $|z| \le \frac{L}{2}$ and $L/(8z^2)$ for $|z| > \frac{L}{2}$ (6)

where z=0 defines the midpoint between the coils (Moran and Kunz, 1962).

The sensitivity function for a two-coil system is plotted in Figure 1.

For a layered model with constant conductivities within the layers, the apparent conductivity response is given as:

$$\sigma_a(z) = \sum_{j=1}^N \sigma_j \cdot \int_{z_j}^{z_{j+1}} S(z - z') \, dz' \tag{7}$$

where *N* is the number of layers and the derivatives used in the Jacobian matrix are easily found as:



Fig. 1. The vertical sensitivity function for a two-coil induction logging tool with a coil separation of 0.50 m.

$$\frac{\partial \sigma_a(z)}{\partial \log\left(\sigma_j\right)} = \sigma_j \cdot \int_{z_j}^{z_{j+1}} S(z-z') \, dz'. \tag{8}$$

With these formulas, the conductivity log inversion problem is solved in exactly the same way as the AEM problem using the inversion formulation of Equations 1 and 2, resulting in conductivities for each of the model layers and estimates of their uncertainty.

Comparisons in the model space

The ModMod comparison is performed by inverting the borehole conductivity log data with the same multi-layer model used in the AEM inversion.

Given the data, the estimates of the data noise and the strength of the vertical regularisation, the layer conductivities are the best estimates of the average conductivity in the depth interval spanned by the layer. By using the same model for the AEM inversion and the borehole inversion, we produce comparable depth averages and can directly compare the values, layer by layer.

We shall express the misfit through a normalised least-squares residual defined as:

$$\Phi_{ModMod} = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \frac{\left(\sigma_j^{aem} - \sigma_j^{log}\right)^2}{var(\sigma_j^{aem}) + var(\sigma_j^{log})}}$$
(9)

where *N* is the number of layers, σ_i^{log} and σ_i^{aem} are the conductivity of the *i*'th layer of the borehole and AEM models, respectively, and $var(\sigma_i^{log})$ and $var(\sigma_i^{aem})$ are the corresponding variances found as the diagonal elements of the posterior covariance matrices of the inversions. The variance of the difference $(\sigma_i^{aem} - \sigma_i^{log})$ is the sum of the variances of the two terms: var $(\sigma_i^{aem} - \sigma_i^{log}) = var(\sigma_i^{aem}) + var(\sigma_i^{log})$ because σ_i^{aem} and σ_i^{log} are uncorrelated.

We have included the variance of both the AEM conductivities and the log conductivities in Equation 9. In the case where either the log model or the AEM model is poorly determined, any misfit between the log and the AEM models must be regarded as irrelevant: no real importance can be attributed to it. Including both variances in the residual ensures that the mismatch flag will not be raised in these situations, i.e. we will not erroneously interpret the discrepancy as significant.

Comparisons in the data space

To be able to carry out a comparison in the data space, we need to compute the forward response of a borehole model. First, the log conductivity data are inverted to find a model, and to ensure that all borehole conductivity variation is accounted for, a 101-layer model is used with constant layer thickness of 1 m. We then compute the AEM forward response, i.e. the data that would be measured by the AEM system if the subsurface had the conductivity structure of the borehole model. In this forward response calculation, we use the same transmitter height and the same gate delay times as in the AEM data closest to the borehole.

In the DatDat comparison, the misfit is expressed through a normalised least-squares residual defined as:

$$\Phi_{DatDat} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \frac{\left(V_i^{aem} - V_i^{log}\right)^2}{var\left(V_i^{aem}\right) + var\left(V_i^{log}\right)}}$$
(10)

where *M* is the number of AEM data, V_i^{log} and V_i^{aem} are the AEM response of the borehole models and the measured AEM data, respectively, and $var(V_i^{log})$ and $var(V_i^{aem})$ are the corresponding variances. Again, the sum of the variances is the variance of the difference $(V_i^{aem} - V_i^{log})$ because the two terms are uncorrelated. The variance of the measured AEM data comes out of the data processing and is used in the AEM inversion, but the variance of the model response of the borehole model needs an explanation.

The measured log conductivity data comes with an uncertainty estimate, and when inverting the borehole data with a 1D model, this uncertainty propagates to the model so that the layer conductivities are associated with an uncertainty. An estimate of this uncertainty comes out of the inversion through the posterior covariance matrix, C_{est} (see Equation 2). When using this model to calculate the AEM forward response, the uncertainty of the layer conductivities propagate to the calculated response. The covariance matrix of the uncertainty of the forward response can be estimated by the following expression:

$$\boldsymbol{C}_{pred} = \boldsymbol{J} \cdot \boldsymbol{C}_{est} \cdot \boldsymbol{J}^T \tag{11}$$

where J is the Jacobian matrix containing the AEM derivatives of the data with respect to the layer conductivities for the borehole model, J^T is the transpose of J and C_{est} is the posterior covariance matrix of the inversion of the borehole data (Taboga, 2012). The resulting matrix, C_{pred} , is the covariance matrix of the uncertainty of the forward response containing the terms $var(V_i^{log})$ in the diagonal.

Relation to traditional practices

In the traditional approach to FID point comparison, the conductivities of the AEM inversion models are most often compared with averages over depth intervals of the measured apparent conductivity of induction logs.

Although this practice seems to be, and has been called, a comparison between (averages of) data and model parameters, conceptually this is a misunderstanding; *data* and *model parameters* simply cannot be compared. A conceptually clear formulation is to say that it is an *implicit* comparison between *models*. By averaging the log apparent conductivity over depth intervals, essentially a *model* is constructed based on the *implicit* assumption that, in a continuous formulation, the sensitivity function of the induction log tool is a Dirac delta function, or in a discrete formulation, that the sensitivity function is a box-car function with unit area, centred on the data point and with a width equal to the sampling density in depth. We say *implicit* because we are not aware of any publication with an explicit formulation that the traditional approach is in essence a model construction.

The model thus constructed is a fair approximation to the model that can be obtained by formal inversion due to the fact that the actual sensitivity function of a two-coil induction log tool (see Figure 1) is non-negative, symmetric and with most of its weight

at the centre. So what is actually achieved by subjecting the induction log apparent conductivity data to a formal inversion? In our opinion, three things: (1) the procedure produces the best model from the log data in a well defined inversion sense (although, as mentioned above, traditional practices will produce a fair approximation); (2) our methodology is conceptually clear; and (3) through a formal inversion formulation, it is possible to keep track of the propagation of uncertainty through all of the FID point comparison procedures. Of these three points we regard the third to be of major importance.

A field example: the BHMAR project

The project

As an example of the application of the ModMod and DatDat comparisons, we shall use data and inversion models of the helicopter-borne transient survey from the BHMAR project. Data were acquired in 2008 with the SkyTEM system (Sørensen and Auken, 2004).

In the southern half of Australia, recent droughts and predictions of a drier future under several climate change scenarios have led to the search for innovative strategies to identify more secure water supplies for regional communities and industries, while also delivering environmental benefits to threatened river systems. It has long been recognised that one of the areas with the greatest potential to contribute water savings in Australia's Murray-Darling Basin is the Menindee Lakes Storages (MLS), located on the lower section of the Darling River in far western New South Wales. The MLS play a significant role in meeting South Australia's water requirements, while also providing the principal water supply storage for the city of Broken Hill. The shallow nature of the Menindee Lakes, which are located in a hot, windy, semi-arid environment, results in the evaporation of up to 700 GL of water per annum (p.a.), with an average loss across the MLS of 420 GL p.a. The opportunity cost of this water evaporating each year (~A\$420 m p.a.), is realised by downstream irrigators, communities and ecosystems. Changing the management of the MLS to provide enhanced water security for Broken Hill, while also reducing the evaporative losses, requires Broken Hill to become less reliant on the MLS.

To address these issues, the BHMAR project was tasked with identifying and assessing managed aquifer recharge (MAR) and/ or groundwater extraction options to secure Broken Hill's water supply, protect the local environment and heritage and return up to 200 GL to the Murray-Darling Basin via the Darling River. To meet the challenge of rapid identification and assessment of potential MAR targets and groundwater resources over a large area (7541.5 km²) within relatively short timeframes (18 months), it was concluded that the only cost-effective method with the ability to resolve key features of the hydrogeological system in the 0-150 m depth range was AEM. In the BHMAR project, the helicopter-borne SkyTEM transient EM system (Sørensen and Auken, 2004) was selected after a rigorous technology assessment exercise (Lawrie et al., 2012a; Christensen and Lawrie, 2012; Smiarowski and Mulè, 2014; Christensen and Lawrie, 2014).

Data acquisition in the BHMAR project involved a phased approach, with investigations initially at a regional scale and subsequent investigations at more local scales as potential groundwater resource and MAR targets were identified and assessed (Lawrie et al., 2012*a*). Investigations involved an integrated, multi-scale hydrogeophysical, hydrogeochemical and hydrogeological systems approach to map and assess nearsurface (<100 m) aquifers and aquitards in unconsolidated alluvial sediments beneath the Darling River floodplain. The study integrated data from the AEM survey with lithological, pore fluid and geophysical data (induction, gamma,and nuclear magnetic resonance) obtained from a 100 borehole drilling program with a total drilling length of 7.5 km.

The AEM survey was instrumental in delineating the key functional elements of the Darling River floodplain hydrogeological system, revealing significant heterogeneity in the subsurface electrical conductivity structure, reflecting a complex geology and variations in groundwater salinity. Significant faulting, warping and tilting are observed to disrupt hydrostratigraphic units (Lawrie et al., 2012b). The study identified several potential MAR and groundwater resource targets (Lawrie et al., 2012b).

The SkyTEM survey

The SkyTEM data were acquired in a standard dual-moment mode with gate centre times for the low moment between 16 and $895 \,\mu s$ and for the high moment between $85 \,\mu s$ and $8.84 \,m s$. With a few exceptions, a flight line spacing of 200 m was used in the entire survey area of ~7500 km², giving a total of ~32 000 line km of data. The SkyTEM system is a dual-moment, calibrated system developed for hydrogeophysical investigations. In the BHMAR survey, the high-moment mode had a peak moment of 125 600 Am². Ramp time was 45 μ s and the base frequency was 25 Hz with a 50% duty cycle. The low-moment mode uses a single transmitter loop turn and a peak current of 40 A. Base frequency in this mode is 222.2 Hz with an on-time of 1 ms and an off-time of 1.25 ms. Ramp time is typically 8 µs. For a more in-depth description of the SkyTEM system, see Sørensen and Auken (2004) and Auken et al. (2009). The SkyTEM data were inverted with a laterally correlated inversion approach (Christensen et al., 2009; Christensen, 2016b) using the Lateral Parameter Correlation methodology (Christensen and Tølbøll, 2009; Christensen, 2016a), including borehole conductivity information from 92 boreholes in the form of log models found from inverting the log data.

A noise model for the conductivity logs

The logging tool used for the induction logs was a two-coil system with a coil distance of 0.50 m. The tool was calibrated using the standard calibration disk, plus a few extra factory calibration disks, to make it possible to construct a calibration curve with several points so that the nonlinear behaviour of the tool in the high-conductivity range could be properly modelled.

To invert the borehole apparent conductivity data of the BHMAR survey area, the noise level of the logs must be estimated. In Ley-Cooper and Davis (2010), the log samples were assigned a basic absolute noise level of 10 mS/m and a relative noise level of 5%, meaning that the variance of the apparent conductivity is:

$$var\left(\sigma_{a,i}^{log}\right) = \left[(10 \text{ mS/m})^2 + \left(0.05 \cdot \sigma_{a,i}^{log}\right)^2 \right].$$
(12)

The conductivity logs were sampled for every 0.025 m, which is very dense compared with the vertical extent of the sensitivity function of ~ 1 m, meaning that data errors are correlated. To compensate for the fact that our inversion program can handle only uncorrelated noise, the noise level must be increased, and we estimated the equivalent uncorrelated noise to be:

$$var\left(\sigma_{a,i}^{log}\right) = \left[(40 \text{ mS/m})^2 + \left(0.10 \cdot \sigma_{a,i}^{log}\right)^2 \right]^2.$$
(13)

ModMod and DatDat comparisons in the BHMAR survey area

For the FID point analyses, a total of 103 borehole logs were available. Of these, 92 were included in the laterally correlated inversion, while 11 were set aside to perform an independent check of the correspondence between borehole logs and AEM models. First, we present the results of the FID point comparison for the 92 logs used in the inversion. We then perform the same analyses on the remaining 11 logs.

To demonstrate the ModMod and DatDat comparisons, plots were made for all 92 boreholes of the BHMAR survey area, illustrating the two comparisons defined above. Only a few examples will be shown here.

Figure 2 shows plots of the ModMod and the DatDat comparisons for borehole BHMAR33–7. An explanation of the plot is given in the figure caption. For BHMAR33–7, the AEM and borehole models match very well with a residual of 1.2.

In Figure 3, plots of the ModMod and DatDat comparisons for borehole BHMAR21–3 are shown; in this case, there is more inconsistency in the FID point comparison, reflected in the



Fig. 2. ModMod and DatDat comparison plots for the BHMAR33–7 borehole. (*a*) Borehole conductivity plotted in dull red with vertical line segments indicating the uncertainty. The 30-layer AEM model closest to the borehole is plotted in black with layer conductivity uncertainties indicated by the grey bars. The 30-layer borehole inversion model is plotted in bright red with layer conductivity uncertainties indicated by the red error bars. The distance between borehole and AEM model is given in the plot title together with the ModMod residual. (*b*) The corresponding plot for the DatDat comparison. AEM data are plotted with black (low moment) and blue (high moment), while borehole response is plotted with red (low moment) and green (high moment). Error bars indicate the uncertainty of the data and response. The DatDat residual is given in the title together with the data residual of the AEM inversion for comparison.



Fig. 3. The figure caption for this figure is identical to the one for Figure 2, except that this figure concerns the BHMAR21-3 borehole.

residual now being 3.8. The top conductive layer is placed at the same depth of ~5 m in both models, but the conductivity of the layer is four times higher in the log model than in the AEM model. The second conductive layer is placed at a depth of ~17 m in the log model, while the AEM model has the conductive layer at ~33 m, again with a lower conductivity than the log model. Notice also that the uncertainty of the borehole model conductivities is very high at the surface. This is caused by the fact that there are no conductivity log samples above ~2.5 m depth. Data in the depth interval 0–2 m were culled for most of the logs to avoid influence from near-surface borehole fittings.

To investigate whether there is a correlation between the residuals of the ModMod comparisons and the distance between the borehole and the AEM model – a larger distance might be connected with a larger misfit – the residuals have been plotted as a function of the distance between the borehole and the closest AEM model (Figure 4). An immediate inspection shows that the correlation is very weak. Likewise, to see if the ModMod residuals are correlated with the data residuals of the AEM inversion – a large degree of discrepancy might be caused by a poorly fitting inversion model – Figure 4 also shows the residuals as a function of the AEM residuals. Again, the correlation appears very weak. A similar investigation to see if the DatDat residuals were correlated with the data residuals

of the AEM inversion gave a similar result (plot not shown). Finally, to investigate whether there is a correlation between the residuals of the ModMod comparisons and depth – a larger depth might be connected with a larger misfit due to the decreasing resolution power of the AEM data – the residuals have been plotted as a function of depth to the layer midpoints when the relative uncertainty of the log model conductivity is lower than 0.2. This condition ensures that only those depth intervals where there are borehole data are used. Once more, the plots show that there is a very weak correlation, if any. A tentative conclusion from these three analyses could be that the misfits between log models and AEM models do not seem to depend on borehole–AEM distance (as long as they are fairly close together), AEM data inconsistency, or depth.

The distributions of the variance-normalised ModMod and DatDat residuals for all boreholes are illustrated in the histograms of Figure 5. The mean of the ModMod residuals is 1.8 with a standard deviation of 0.7 and a maximum value of 5. The mean of the DatDat residuals is 6.3 with a standard deviation of 4.7. The fact that the ModMod residuals are of the order of 1 shows that, overall, there is good consistency between the borehole models and the AEM models, given the uncertainty of the AEM inversion. The DatDat residuals are somewhat higher, but the mean value of 6.3 must be compared with the mean value of the



Fig. 4. (*a*) ModMod residuals as a function of the distance between the borehole and the closest AEM sounding. (*b*) ModMod residuals as a function of the data residual of the AEM inversion for the AEM sounding closest to the borehole. (*c*) ModMod residuals as a function of depth to the layer midpoint. (*d*) Uncertainty-normalised relative ModMod residuals as a function of depth to the layer midpoint. In all plots, there is no correlation between the plotted parameters.

Fig. 5. Histograms showing the distribution of the residuals listed in Table 1. (a) ModMod residual; (b) DatDat residual; and (c) residual of the AEM inversion.

data residual in the AEM inversion, which is 2.9, meaning that they are of the same order of magnitude, so overall there is a good consistency between the measured data and the predicted data from the borehole models. It is worth noting that using the AEM 30-layer model, instead of the 101-layer model, to calculate the AEM forward response in the DatDat comparison gave practically identical results (comparison not shown here).

Estimating a functional relationship in the ModMod comparison

Figure 6 shows a cross-plot of the values of the layer conductivities of the 92 borehole models used in the inversion and the AEM models in a log-log plot with pertinent error bars. The plotted values lie close to the straight line, indicating an identity mapping, as should be expected from the fact that the values of the average residuals are all small numbers.

To avoid coupling between the conductivity log measurements and the top fittings of the borehole, log values for the top $\sim 2 \text{ m}$ were discarded for all logs. Furthermore, the maximum depth for the most of the boreholes was less than the 200 m spanned by the 30-layer model. As a consequence of this, and the fact that the sensitivity function of the log tool has a vertical extent of only $\sim 1 \text{ m}$, the model conductivities of the layers without any log data are determined solely by the prior model of the inversion, and their uncertainty will be very high. These layers were excluded in the cross-plot in Figure 6 because the uncertainty of the layer log(conductivity) was larger than 0.2.

In many of the logs, there is no data coverage in the top few metres, and consequently, through the regularisation, the conductivities of the near-surface layers of the borehole model are determined primarily by the background model. This is the cause of the difference seen for early gates in the DatDat comparison.

A linear regression in log space was performed to find a functional relationship between the two sets of conductivities. In the regression analysis, the uncertainties on both borehole layer conductivities and AEM layer conductivities have been taken into account, i.e. we have found the parameters a and b that minimise the residual:

$$\Phi_{err}^{ModMod} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \frac{\left[\log \sigma_i^{log} - (a \cdot \log \sigma_i^{aem} + b)\right]^2}{var\left(\log \sigma_i^{log}\right) + a^2 \cdot var\left(\log \sigma_i^{aem}\right)}}$$
(14)

 $\log \sigma_{log} = a \cdot \log \sigma_{aem} + b = 1.0245 \cdot \sigma_{aem} - 0.0412 \implies$ $\sigma_{log} = e^b \cdot (\sigma_{aem})^a = 0.9596 \cdot (\sigma_{aem})^{1.0245}$ (15)

where N is the total number of layers over all models fulfilling

the selection criterion. The regression has been carried out using

the MATLAB script: York_fit.m (© Travis Wiens, 2010, travis.

mlfx@nutaksas.com) that is freely available on the MathWorks

website (York et al., 2004). The plots are seen in Figure 7, and the

with a residual of $\Phi_{err}^{ModMod} = 1.6783$.

regression parameters are:

In Figure 6, a thin cyan line indicates the identity line between σ_{aem} and σ_{log} , while the regression line, taking the uncertainties into account, is indicated by a thicker red line. It is seen that the residual is of the order of unity, indicating that the fit is good and the functional relationship is close to the identity mapping.

In traditional FID point comparison, the variances of σ_{aem} and σ_{log} are not taken into account, and the regression becomes a simple unweighted linear regression. To compare this practice with the one we suggest in this paper, we also carried out the unweighted regression, which gave the result: $\sigma_{log} = 1.5682 \cdot (\sigma_{aem})^{0.9591}$ and a residual of $\Phi_{unweighted}^{ModMod} = 0.5348$. This regression line is also plotted in Figure 6. In this case, the difference between the two approaches is small.

In the case where uncertainty is neglected, the correlation coefficient, R^2 , traditionally used in linear regressions, is given by:

$$R^{2} = 1 - \frac{SS_{err}}{SS_{tot}} = 1 - \frac{\sum [y_{i} - (ax_{i} + b)]^{2}}{\sum (y_{i} - \bar{y})^{2}}$$
(16)

where SS_{err} is the unnormalised residual defined above, SS_{tot} is the total variability of the data and \bar{y} is the mean value of y. Equation 16 is the expression relevant for the unweighted regression, and we achieved a value of $R_{unnorm}^2 = 0.5971$.

In the case where uncertainty on both x and y is taken into account, we must derive a generalisation of the above formula. The generalised residual term is already defined in Equation 9 so, in the same way, we need to normalise the expression for the total variability with its variance. It is easy to prove that the covariance between y_i and \bar{y} is zero, so the variance of the difference is the sum of the variances:

$$var(y_i - \bar{y}) = vary_i + var\bar{y} = vary_i + 1 / \sum_{i=1}^{N} \frac{1}{vary_i}.$$
 (17)

Fig. 6. Cross-plots on a logarithmic scale of the AEM model conductivity as a function of the borehole model conductivity. Only the borehole model layers for which the relative uncertainty of the conductivity is less than 0.2 are included. (*a*) In the log-log plot, the uncertainties of both parameters are indicated by error bars. (*b*) The identity mapping is shown with a thin cyan line, while the least-squares linear fit, taking uncertainties on both parameters into account, is indicated with a thicker red line. The line obtained in an unweighted regression is shown in green. The error bars have been omitted for clarity.

Fig. 7. Cross-plots on a logarithmic scale of the AEM forward model response of the borehole model as a function of the AEM data. (*a*) In the log-log plot, the uncertainties of both parameters are indicated by error bars. (*b*) The identity mapping is shown with a thin cyan line, while the least-squares linear fit, taking uncertainties on both parameters into account, is indicated with a thicker red line. The error bars have been omitted for clarity.

The variance-normalised correlation coefficient will therefore be given by:

$$R_{norm}^{2} = 1 - \frac{\sum \left\{ \left[y_{i} - (ax_{i} + b) \right]^{2} / \left[var y_{i} + a^{2} \cdot var x_{i} \right] \right\}}{\sum \left\{ \left(y_{i} - \bar{y} \right)^{2} / \left[var y_{i} + 1 / \sum (1 / var y_{i}) \right] \right\}}.$$
(18)

Using this formula we find for the ModMod comparison $R_{norm}^2 = 0.9915$, a value much higher than the one obtained in the case of unweighted regression.

Estimating a functional relationship in the DatDat comparison

Figure 7 shows a cross-plot of the values of the measured AEM data and the data predicted from the borehole models of the

92 borehole models used in the inversion in a log-log plot with the pertinent error bars. It is clear that the plotted values lie close to the straight line, indicating identity between them, as should be expected from the fact that the values of the average residuals are all small numbers.

In the same way as for the ModMod comparison, a linear regression in log scale has been performed to find a functional relationship between the AEM data and the forward response of the borehole models. In the regression analysis, the uncertainties on both parameters have been taken into account, i.e. we have found the parameters a and b that minimise the residual:

$$\Phi_{err}^{DatDat} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \frac{\left[\log V_i^{log} - (a \cdot V_i^{aem} + b)\right]^2}{var\left(\log V_i^{log}\right) + a^2 \cdot var(\log V_i^{aem})}}$$
(19)

where *M* is the total number of data over all models. The optimum parameters of the functional relationship and their uncertainties were found to be: $a = 0.966 \pm 0.00071$ and $b = -0.471 \pm 0.0151$, so that:

$$\log V_{log} = a \cdot \log V_{aem} + b = 0.966 \cdot V_{aem} - 0.471 \implies$$

$$V_{log} = e^b \cdot (V_{aem})^a = 0.6243 \cdot (V_{aem})^{0.966}$$
(20)

with a residual of $\Phi_{err}^{DatDat} = 3.1$.

In Figure 7, a thin cyan line indicates the identity line between V_{aem} and V_{log} , while the regression line, taking the uncertainties into account, is indicated by a thicker red line. The residual of 3.1 is of the order of unity, indicating that the fit is good and the functional relationship is close to the identity mapping. Using Equation 18, we find for the DatDat comparison $R_{norm}^2 = 0.99064$.

Results of the ModMod and the DatDat comparisons for the independent boreholes

In modern inversion practices, the information from the borehole conductivity logs is included as constraints in the inversion of the AEM data. The borehole logs represent valuable information that should naturally be included in the inversion to obtain the best possible models. The borehole log information will spread to the adjacent models through the lateral correlation of the inversion, and experience from this project shows that, with the lateral correlation strengths used in the final inversion, the borehole information affects the inversion models within a radius of 50–300 m from the borehole. It follows that the misfit between the borehole log and the closest AEM inversion model will become smaller when borehole information is used in the inversion than when it is not.

In the comparisons of the previous section, the AEM models were calculated using such a correlated inversion, taking the borehole information into account, and the results showing little inconsistency between the AEM models and the borehole models should be seen in that perspective: the borehole models and the AEM models are not independent. However, in the BHMAR survey area, 11 boreholes were drilled and logged before the BHMAR survey and not included in the AEM inversion, and we can therefore establish a truly independent comparison between the AEM inversion results and the 11 borehole conductivity logs. All analysis procedures are the same as the ones used in the previous sections for the BHMAR boreholes, so only a summary of the results is included here.

Figure 8 shows plots of the ModMod and the DatDat comparisons for borehole GW36812. An explanation of the plot is given in the caption of Figure 2. For the GW36812

Fig. 8. The figure caption for this figure is identical to the one for Figure 2, except that this figure concerns the GW36812 borehole.

there is a very high degree of consistency in the FID point comparison.

In Figure 9, plots of the ModMod and the DatDat comparisons for borehole GW36891 show there is more inconsistency in the FID point comparison. The AEM model does register the high conductivity layer seen in the log at ~10 m, although it is placed somewhat deeper, but the AEM model indicates higher conductivity below a depth of 100 m.

Similar to the boreholes included in the BHMAR inversion, the ModMod residuals are uncorrelated with the distance between the borehole and the AEM sounding and with the AEM inversion residual (plots not shown).

The fact that the ModMod residuals are of the order of unity shows that, overall, there is good consistency between the borehole models and the AEM models. The DatDat residuals are somewhat higher, but the mean value of 4.5 must be compared with the mean value of the data residual of the AEM inversion, which is 2.1, meaning that they are of the same order of magnitude, so overall there is a good consistency between the measured data and the predicted data from the borehole models.

All residuals and their mean, standard deviation and median values are listed in Table 1.

Estimating a functional relationship in the ModMod comparison for the independent boreholes

Figure 10 shows a cross-plot of the values of the layer conductivities of the 11 borehole models not used in the inversion and the AEM models in a log-log plot with the pertinent error bars. Most of the plotted values lie close to the straight line, indicating identity between them, as should be expected from the fact that the values of the average residuals

are all small numbers. Closer inspection reveals that when layer conductivities are in the middle range of 40–400 mS/m, there is a tendency for σ_{log} to be higher than σ_{aem} . This could be an

Fig. 9. The figure caption for this figure is identical to the one for Figure 2, except that this figure concerns the GW36891 borehole.

expression of the fact that in the depth range where conductivity log data are present, the depth resolution of AEM data is poorer than the depth resolution of the conductivity log data, and in the regularised inversion with vertical smoothness constraints, the AEM models will therefore be smoother. It can also be seen that for layer conductivities above 1000 mS/m, σ_{aem} is clearly higher than σ_{log} . At the moment, we do not have a good explanation for this behaviour, but naturally it affects the functional relationship derived below shifting the regression line downwards. For the same reason as before, layers where the uncertainty of the layer log(conductivity) of the borehole model were larger than 0.2 have been omitted.

The parameters minimising the residual in Equation 14 and their uncertainties were found to be: $a = 1.1418 \pm 0.0461$ and $b = -1.1728 \pm 0.2797$, so that:

 Table 1. ModMod and the DatDat residuals for all 11 boreholes not included in the inversion, together with the data residual of the AEM inversion.

'Dist' is the distance between the borehole and the nearest AEM location. 'StdDev' is the standard deviation.

Label	Dist	ModMod	DatDat	AEM_ResDat
GW36807	73	0.9	2.6	1.3
GW36812	30	1.6	2.3	2.7
GW36815	82	1.2	1.3	1.7
GW36836	49	1.7	1.9	2.4
GW36837	63	1.6	3.5	3.0
GW36838	146	1.7	7.9	2.0
GW36891	28	2.0	4.8	3.0
GW87790	60	0.6	2.2	1.4
GW878001	58	1.0	4.5	1.7
GW87801	38	1.2	5.3	1.6
EX1	100	2.1	13.1	2.8
Statistical parameters				
Mean		1.4	4.5	2.1
StdDev		0.5	3.3	0.6
Median		1.6	3.5	2.1

Fig. 10. The figure caption for this figure is identical to the one for Figure 6, except that this figure concerns the 11 boreholes not used in the inversion.

Fig. 11. The figure caption for this figure is identical to the one for Figure 7, except that this figure concerns the 11 boreholes not used in the inversion.

$$\log \sigma_{log} = a \cdot \log \sigma_{aem} + b = 1.1418 \cdot \sigma_{aem} - 1.1728 \Rightarrow$$

$$\sigma_{log} = e^b \cdot (\sigma_{aem})^a = 0.3095 \cdot (\sigma_{aem})^{1.1418}$$
(21)

with a residual of $\Phi_{err}^{ModMod} = 1.8038$.

In Figure 10, a thin cyan line indicates the identity line between σ_{aem} and σ_{log} , while the regression line, taking the uncertainties into account, is indicated by a thicker red line. It is seen that the residual is of the order of unity, and while the power of the functional relationship is close to unity, the factor is smaller than unity due to the skewed distribution of the higher conductivities mentioned above. The correlation coefficient for the ModMod comparison was found to be $R_{norm}^2 = 0.9832$.

Estimating a functional relationship in the DatDat comparison for the independent boreholes

The DatDat comparison for the 11 independent borehole logs was carried out in exactly the same way as the one for the 92 borehole logs included in the inversion and the behaviour is very similar. As such, we only show the results here and you must refer to the previous section for more details.

Figure 11 shows a cross-plot of the values of the measured AEM data and the data predicted from the borehole models in a log-log plot with the pertinent error bars. The optimum parameters of the functional relationship and their uncertainties were found to be: $a = 1.0344 \pm 0.0015$ and $b = 0.8322 \pm 0.0331$, so that:

$$\log V_{log} = a \cdot \log V_{aem} + b = 1.0344 \cdot V_{aem} + 0.8322 \implies V_{log} = e^b \cdot (V_{aem})^a = 2.2984 \cdot (V_{aem})^{1.0344}$$
(22)

with a residual of $\Phi_{err}^{DatDat} = 2.85$.

In Figure 11, a thin cyan line indicates the identity line between V_{aem} and V_{log} , while the regression line, taking the uncertainties into account, is indicated by a thicker red line. It is seen that the residual is of the order of unity, indicating that the fit is good and the functional relationship is close to the identity mapping.

The correlation coefficient for the ModMod comparison $R_{norm}^2 = 0.9984$. If we use the formula with the unweighted sums, we find $R^2 = 0.9999$.

Discussion and conclusion

We have presented two novel and self-consistent ways of performing a comparison between AEM inversion results and borehole conductivity logs: comparison in the model space and in the data space. We find these comparisons to be a significant improvement over the traditional FID point comparison where AEM model conductivities are compared with averages of apparent conductivity data from a borehole conductivity log. Although the two parameters both have the dimension of [S/m], they belong to different conceptual classes: model parameters and data.

The ModMod and DatDat comparisons require the inversion of the induction log apparent conductivity data, including an estimate of the posterior covariance matrix. However, in the lowfrequency approximation valid for the majority of conductivity log data, the inversion presents no difficulties at all.

The two self-consistent comparisons, the ModMod and DatDat comparisons, are easy to implement, and the MATLAB linear regression script that incorporates uncertainty on both variables is freely available. The two methods require an AEM inversion code capable of forward modelling and inversion that will also produce an estimate of the posterior covariance matrix, but most modern inversion codes fulfil this criterion. If a laterally correlated inversion is performed, constraints are implemented between AEM models and it would be straightforward and internally consistent to include the models derived from inverting the borehole logs.

The DatDat comparison requires the simple inversion of the conductivity log data plus the calculation of the AEM sensitivities; it does not refer to any of the intricacies of AEM inversion, like convergence control and vertical and horizontal constraints, which is typically where inversion codes differ. DatDat analyses are therefore directly comparable across inversion platforms and programs. The ModMod and DatDat analyses were applied to the 92 boreholes of the BHMAR project involving the AEM inversion models obtained through vertically and laterally correlated inversion, including the 92 boreholes modelled. The comparisons showed a high degree of consistency between the AEM models and the borehole models, but since the borehole models were involved as constraints in the inversion, the high degree of consistency mostly shows that the inversion approach was sound. However, the fact that the residuals of the AEM models closest to the boreholes were of the order of unity also shows that large discrepancies between AEM and borehole models were absent.

An independent analysis was carried out involving 11 borehole conductivity logs that were not included as constraints in the overall AEM inversion. The comparisons showed a high degree of consistency, which further strengthens the conclusions that: (1) the BHMAR inversion was sound, (2) that it was justified to include the borehole conductivity logs as constraints in the inversion, and (3) that large discrepancies between AEM and borehole models were absent. A value of 0.94 was obtained for the variance-normalised general R^2 measure for the bores distributed throughout the project area that were not used as inversion constraints (Lawrie et al., 2012*a*, 2015).

As mentioned in the Introduction, there are two main issues to be considered when assessing possible inconsistencies between borehole conductivity logs and the models resulting from AEM inversion. First, the huge scale difference between the sensitivity volumes of the induction tool and an AEM system suggests a situation where many irrelevant matches/mismatches must be expected in a log-AEM comparison and there is no way to discern whether the results of an individual comparison are relevant or not. Second, there is a bias in the placement of the borehole locations towards locations that carry the most potential for the subject of the investigation, e.g. the groundwater potential. However, the scale issue is a long and complicated discussion worthy of its own research effort and beyond the scope of this paper.

It is worth noting that even though the residuals in the DatDat comparisons over the 92 boreholes included in the inversion and the 11 boreholes used in the independent test were acceptably small, the factors of the functional relationships in Equations 20 and 22 are not close to unity: the values are 0.62 and 2.30, respectively. This shows the importance of including the uncertainty of all parameters in the comparison. It also indicates that a practice of comparing AEM data and forward response of the borehole models with the purpose of calibrating the AEM data should be approached with great care. We are certain that the SkyTEM data are not wrong with a factor of 0.62 or 2.30. In our opinion, the cause of the fairly high factors is rooted in the scale issue. However, the forward modelling exercise can be trusted to disclose whether the overall structure of the data are the same.

Our analyses have been centred around comparing borehole information with AEM surveys, but the methodology offers itself equally well to both AEM and ground EM geophysical methods. We consider the ModMod and the DatDat comparison to be a better practice than the traditional FID point comparison.

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