# Quantification of modeling errors in airborne TEM caused by inaccurate system description

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# ABSTRACT

Being able to recover accurate and quantitative descriptions of the subsurface electrical conductivity structure from airborne electromagnetic data is becoming more and more crucial in many applications such as hydrogeophysical and environmental mapping, but also for mining exploration. The effect on the inverted models of inaccurate system description in the 1D forward modeling of helicopter time-domain electromagnetic (TEM) data was studied. The most important system parameters needed for accurate description of the subsurface conductivity were quantified using a nominal airborne TEM system and three different reference models to ensure the generality of the conclusions. By calculating forward responses, the effect of changing the system transfer function of the nominal airborne TEM system was studied in detail. The data were inverted and the consequences of inaccurate modeling of the system transfer function were studied in the model space. Errors in the description of the transfer function influence the inverted model differently. The low-pass filters, current turn-off, and receiver-transmitter (Rx-Tx) timing issues primarily influenced the early time gates. The waveform repetition, gate integration, altitude, and geometry mainly influenced the late time gates. Depth of investigation is highly model dependent, but in general the early times hold information on the shallower parts of the model and the late times hold information on the deeper parts of the model. Amplitude, gain, and current variations affect the entire sounding and therefore the entire model. The results showed that all of these parameters should be measured and modeled accurately during inversion of airborne TEM data. If not, the output model can differ quite dramatically from the true model. Layer boundaries can be inaccurate by tens of meters, and layer resistivities by as much as an order of magnitude. In the worst cases, the measured data simply cannot be fitted within noise level.

# **INTRODUCTION**

For groundwater and environmental applications the airborne electromagnetic (AEM) data are used quantitatively and as a consequence the reliability of the model parameters obtained by inversion is crucial. This calls for high-quality data, accurate forward modeling of the systems, and precise and robust inversion. Here, we focus on accurate forward modeling using synthetic models to quantify the effect of inaccuracies in the system description.

In sedimentary areas the electrical conductivity is mainly dependent on the salinity of the groundwater (i.e., the groundwater quality) and the clay content of the subsurface (i.e., the aquifer conditions and protection level) (Kirsch, 2006; Siemon et al., 2009). As a consequence, in large-scale groundwater surveys AEM methods are most often chosen because they offer an efficient tool to investigate the conductivity structure over large areas in a reasonable time and at relatively low costs.

# Airborne time-domain electromagnetic systems

The application of geoelectrical and surface electromagnetic methods has a long tradition in groundwater and environmental exploration. However, AEM was introduced for mineral exploration and — compared with that — airborne groundwater exploration is a relatively new application. Siemon et al. (2009) give a comprehensive overview of AEM methods used for groundwater applications.

Various AEM systems are used for airborne groundwater and environmental investigations, including rigid-beam helicopter-borne

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frequency-domain systems, fixed-wing time-domain systems, fixed-wing frequency-domain systems, and several helicopterborne time-domain systems (Fountain, 2008). In the last decade development has focused on the helicopter time-domain systems because they have superior depth penetration compared with the frequency-domain systems. However, most time-domain electromagnetic (TEM) systems have less resolution in the near-surface compared with the frequency-domain systems. The helicopter timedomain systems are the focus of this paper.

Recently developed helicopter TEM systems are the AeroTEM, Hoistem, VTEM, HeliGEOTEM, and SkyTEM systems (an overview is presented by Allard, 2007). These systems were originally designed for mineral exploration. The SkyTEM system represents an exception because it was purposely designed for mapping of geological structures in the near-surface for groundwater and environmental investigations. Helicopter systems carry the transmitter (Tx) loop as a sling load beneath the helicopter. In the Tx loop an electric current is abruptly terminated, causing a change of the primary magnetic field, which in turn induces currents to flow in the ground. Because of ohmic loss the currents decay and, in general, diffuse downward and outward in the subsurface. The change over time (decay rate) of the secondary magnetic fields from these currents is picked up by an induction coil, typically located near the Tx frame. In most cases, two perpendicular receiver (Rx) coils pick up the inline field (x-component) and the vertical field (z-component). For groundwater exploration the vertical field holds by far most of the information because the geological structures are predominantly horizontal. The large horizontal currents induced in the ground mean that the inline fields are small when there is only a small offset between the Tx loop and the Rx coil.

#### Modeling airborne TEM systems

Auken et al. (2007) showed some of the most important parameters that influence the system transfer function of airborne TEM systems; namely, the altitude of the transmitter-receiver (Tx-Rx), the waveform, and the Tx-Rx timing. In addition to these factors there is the general geometry of the Tx-Rx, the waveform repetition, the integration of the signal over the width of the time gate, and the lowpass filters in the Rx system.

# Table 1. Central parameters defining a nominal airborneTEM system used for the examples in this paper

Transmitter			
x,y,z (m)	0, 0, -30		
Turn-on	100 µs		
Turn-off	3 µs		
Area	500 m <sup>2</sup>		
Current	1 A (normalized)		
Base frequency	25 Hz, 10 ms on, 10 ms off		
Receiver			
x,y,z (m)	0, 0, -30		
Filters	None		
First gate-center time	13 µs		
Last gate-center time	8 ms		
Gate distribution	10 gates per decade in time		

Bias and leveling problems also present real and serious problems with many airborne systems and a lot of effort is put into the removal of these effects during data collection and data processing. This paper will not discuss these issues and we thereby take as the starting point that no leveling or bias issues are left with the data.

#### MODELING EXAMPLES

#### The nominal system and models

All airborne TEM system are different and they all have their own characteristics and design considerations. For this reason, and to keep things general, we will model the response of a nominal system. The base system is a sling-load, central-loop system with the Tx towed 30 m above the ground. The Tx is a 500-m<sup>2</sup> circular loop transmitting a trapezoidal waveform in a 50% duty cycle and a turn-off time of 3  $\mu$ s. The first gate-center time is 13  $\mu$ s after the beginning of turn-off and the last gate is at 8 ms. The nominal setup is summarized in Table 1.

We assume that there are no bias or calibration issues in the data and we do not consider data noise. Noise levels depend heavily on local factors and a given noise model is generally wrong for specific cases. We have assigned 5% uniform noise to all gates used in the inversion. By setting the last usable gate at 8 ms, we have indirectly assumed a fairly low noise level. Also, the first gate is quite early at 13  $\mu$ s. In that sense, this system and the data can be regarded as slightly optimistic. However, many of the systems currently flying are being developed further in the hope of achieving these parameters.

For each of the different examples we will use one of the three different models as presented in Figure 1. Model a is a low-contrast model with a 30-m layer in the top representing till overlying an aquifer, which in turn is underlain by a clay layer. Model b has higher contrasts and the model is dominated by a very conductive last layer with higher resistivities above. This is meant to resemble an aquifer model with salt intrusion at depth. Model c is a double ascending model representing clay and till overlying sands, chalk, or some other resistive bedrock.

In each of the examples we will first show the effect that the parameter in question has on forward responses. Then we will assume a realistic inaccuracy on the specification of the parameter. Inverting the data, we show the effect on the final output models. All of the inversions presented have been given the true model as the starting model. The deviations are therefore the minimum changes in the inverted model required to fit the inaccurate forward responses. If the inversions had been started with a homogenous half-space, as is usually the case with field data, the deviation from the true model would likely be larger. The model (a, b, or c) chosen for the individual examples is the model that gives the most significant result with respect to that particular example.

Forward modeling and inversion of AEM data are not within the scope of this paper. The inversion algorithm itself is described in detail in Christiansen and Auken (2008). The same algorithm implements the laterally and spatially constrained inversion concepts described in Auken et al. (2005) and Viezzoli et al. (2008). The theory of the forward algorithm is given in Ward and Hohmann (1988). The algorithm implements a complete modeling of a piecewise linear current ramp, low-pass filters, gate integration, and as inversion parameters the system altitude and for some systems the system geometry.

# Example 1 — Filters

All airborne TEM systems have low-pass filters in the Rx coil and the Rx electronics and the amplifier and coil systems themselves act as low-pass filters. The filters are put in place to reduce the ambient noise at late time gates and to reduce coherent noise from radio transmitters. The cut-off frequency and slope of the filters vary significantly and, regrettably, the operating companies often consider this as proprietary information.

In the following examples we approximate the amplification versus frequency characteristic of the filter by that of the Butterworth filter (Bianchi and Sorrentino, 2007) and calculated as described in Effersø et al. (1999).

If we assume an ideal impulse response system, the low-pass filtered output R as a function of time t is the convolution between the filter F and the sum of the primary signal P and the secondary signal S (the earth response),

$$R(t) = \int_{-\infty}^{\infty} [P(t') + S(t')]F(t - t')dt'.$$
 (1)

Thus, the primary field and the earth response will charge up the filters, which in turn release the charge-up in the off time. The primary signal itself is often compensated in different ways at the Rx, reducing the charge-up according to the equation above. Applying no compensation at all and having a low cut-off frequency results in a filter contribution that at early times can be orders of magnitude higher than the earth response. This is demonstrated in Figure 2a, where the effects of different first-order filters without any compensation of the primary field entering the filter are shown. The distortion of the decay curve reaches beyond 100  $\mu$ s and for the 5-kHz filter as far as 700  $\mu$ s. The effect of a low-pass filter at 200 kHz is limited. Filters with higher orders affect the early gates more and has a measurable effect on the later gates. However, it is clear that even a moderate low-pass filter can distort the signal by orders of magnitude at early times.

Systems with a central-loop configuration will most often have some sort of compensation coil (also known as bucking coil) to min-

imize (ideally remove) the primary field at the Rx, which is placed at the maximum coupling position. This reduces the energy that is transferred from the primary signal to the filter. High-altitude measurements are used on top of this to remove primary signal not fully compensated by the bucking coil. Another way of compensating a filter is to introduce a time shift. A time shift is a first-order approximation of the filter effect on the data.

Having a bucking coil and trying to compensate the primary field is not trivial because different processes influence the compensation severely.

 At high altitudes, the measuring conditions are different than those at survey altitude (e.g., temperature varies significantly), which means that amplifier characteristics change from the moment the compensation is determined to when it is applied at survey altitude.

- Distortions of the frame geometry will influence the area and positioning of the bucking coil relative to the Tx coil, which affects the compensation.
- 3) The Tx and the bucking coil have different electrical properties (capacitance C, self-induction L, and resistance R), which means that during the turn-off, the bucking coil only compensates ideally in the avalanche part (when the transistors controls the current flow). When the turn-off enters the free decay, the coils will decay independently and the compensation is no longer perfect.
- 4) At high altitudes there is no earth response, so the filter chargeup depends solely on the primary signal. At survey altitudes the filter will also be charged by the earth response. Thus, subtracting the high-altitude measurements does not produce a filterfree response, and ideally only the primary field part of the filter is removed. However, the compensation has to be extremely accurate and stable because the magnitude of the filtered primary response often will be large compared with the earth response at early time-gates.

The charge-up of the filter resulting from the primary signal will be reduced by the bucking coil combined with high-altitude subtraction and other techniques such as deconvolution. However, it is not likely that this subtraction is perfect and it is very difficult to estimate the amount of residual signal after the subtraction has taken place.

Figure 2b shows the effect of only a partial compensation of the primary field using a 15-kHz first-order low-pass filter. It is clear that even 1% of residual primary field will introduce a large effect on the measured response. At 0.1% and 0.01% primary residual, the filter distortion becomes more and more dominated by the charge-up and release of the earth response itself.

First, let us assume we were inverting a data set that was measured with a 70-kHz Butterworth-type filter (the red curve in Figure 2a), and there has been no compensation of any kind of the filter effect. However, the modeled filter frequency is slightly off. This is shown by the green and red curves in Figure 3a, where the filter cut-off frequency is off by 5 kHz. The comparison with the true model (model a in Figure 1) clearly shows the consequences on the shallow to inter-



Figure 1. The three different models used for experiments in the following sections.

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mediate part of the model. Overestimating the cut-off frequency has the most pronounced effect on the recovered model, introducing an artificial thin shallow conductor. The overall data fit (Figure 3b) is within the data noise, but a slightly poorer fit is noticeable for the early times where the filter effect is most pronounced.

If the cut-off frequency of the low-pass filters is even more distant from the correct one, data cannot be fitted at all and the only way to obtain a reasonable fit is to delete the early gates.

Now, as the second filter example we will still assume a 70-kHz filter, but the primary charge-up of the filter has been fully compensated by the bucking coil and subtraction of high-altitude measurements (i.e., the orange curve in Figure 2b but with a 70-kHz filter). Inverting this data set, assuming that no filter at all has been applied, results in the blue model in Figure 3a. Data fits for this model (blue curve in Figure 3b) are near perfect and give no indication of an erroneous model.

Often AEM systems have filters of higher order than one and/or of lower cut-off frequency. To model their low-pass filter characteristics correctly while inverting is mandatory if quantitative information about shallow and intermediate-layer resistivity and boundaries is sought. Failure to do that results in near-surface artifacts and/or data for the earlier gates that cannot possibly be fitted as well as loss of potentially valuable near-surface information.



Figure 2. The effect on the forward response of low-pass filters on model a. (a) The effect of first-order filters without any compensation of the primary signal is shown. The black curve shows no filters, the purple curve shows the effect of a 200-kHz filter, the red curve shows the effect of a 70-kHz filter, the green curve is a 15-kHz filter, and the blue curve is a 5-kHz filter. The huge overshoot at early times is caused by the charge-up of the filter by the primary response from the Tx turn-off. (b) The effect of a partial compensation of the primary field for a 15-kHz first-order filter is shown. The purple curve is without compensation (100% primary — same as in panel (a), the green curve is 0.1% primary left, and the orange curve is 0.01% primary left, and the black curve is without a filter.

Most of the airborne systems currently in operation apply lowpass filters in the range discussed above. The possible effect of a guess, as qualified as it may be, is clearly seen in the above example.

# Example 2 — Waveform repetition

TEM responses are often modeled using only one repetition of the waveform, but all TEM systems operate with alternating polarity of current pulses, primarily to remove harmonics from the power line frequency. In practice, this means that there is less response than actually modeled using only one waveform compared with modeling two or more pulses. For high-resistivity models this is not a problem because the effect is dominating at late times. But for deep low-resistive layers the effect of previous current pulses is definitely recognizable.

In Figure 4a the red curve shows the forward response corresponding to model b in Figure 1 when modeling only one polarity (+; 0) of the waveform, and the green curve shows the forward response when modeling the full cycle (+; 0; -; 0).

A close look at Figure 4b at the late times shows a clear difference in forward responses (note that the error bar is 5%). The changing polarity of the transmitted waveforms causes the earth response to decrease.

The effect of inverting the two-waveform repetition response assuming only one repetition is seen in Figure 5. The forward data from the inverted model fit almost perfectly to the observed data, but the increased voltage response due to using only one waveform causes the depth to the deep good conductor to be shifted 20 m



Figure 3. The effect on the inverted models having frequency errors on model a in Figure 1. (a) The result of the inversion, where the black curve shows the true model. The red curve shows the effect of underestimating the true filter frequency (65 instead of 70 kHz), and the green curve is an overestimation of the true filter frequency (75 instead of 70 kHz). The blue curve shows the inverted model if the primary charge-up of a 70-kHz filter has been fully compensated but modeled without a filter. (b) The fit to the observed data (black error bars) is shown with the same color code.

downward. The middle high-resistivity layer is also overestimated, but this is much less important because the 500- $\Omega$ m layer in the true model is very poorly resolved in the first place.

# Example 3 — Tx current turn-off

Variation in the turn-off time of the Tx current mainly affects the early time-gates whereas the late time-gates are hardly affected at all. Two things cause the effect: first, the induction in the subsurface is different for a different turn-off ramp; second, the distance, in time, between the first gate and the end of the ramp gets smaller for larger turn-off times. The latter effect is by far dominating and is clearly seen as a raised signal level for the longer turn-off times.

The differences in the forward responses due to inaccurate modeling of turn-off-times are seen in Figure 6. Most modern systems are capable of accurately recording the turn-off time and for some systems the turn-off time is provided for each flight or even for each sounding.

Let us now assume that we model the turn-off time as 3  $\mu$ s as given for the nominal system description, but actually the real turn-off time for that sounding is 4 or 7  $\mu$ s. The results from inverting these two data sets are shown in Figure 7.

The inverted model from an erroneous assumption of the turn-off time of only 1  $\mu$ s is shown with blue in Figure 7a. As expected, the inverted model is only affected in the shallow part. The few percent error of the forward response caused the first boundary to shift 6.7 m up, which can indeed be important for accurate hydrological modeling. Resistivities are in this particular case unaffected. If the assumed turn-off time is even further from the truth, the inverted models are affected quite heavily, introducing a thin conductor at the surface to comply with the elevated response arising from the erroneous description of the turn-off time.

# Example 4 — Timing

AEM systems can also have problems with an absolute determination of the relative time shift between the Tx and the Rx system or

the time shift is not properly defined. Let us now model the effect of not taking these shifts into account. This is shown in Figure 8. A  $-6-\mu s$  shift means that the Tx pulse is moved 6  $\mu s$  backward in time (i.e., the actual gate-center time of the first gate is 6  $\mu s$  later than the nominal value). The effect on the forward responses is very clear at early times, but it can be identified even at 200–300  $\mu s$ .

Let us again assume that we model the timing as indicated in the nominal system, but actually the measured data are shifted -3 or even  $-6 \ \mu$ s. Inverting these data gives us the models presented in Figure 9a. A timing error of just  $-3 \ \mu$ s virtually removes the resistivity contrast between layer 1 and layer 2 and we have a two-layer model. On the data fit (Figure 9b) we see that there actually is a small misfit to the first one or two gates, but without any other indication that there might be a problem with the timing, this kind of misfit is very hard to recognize at individual soundings. An error of 6  $\mu$ s completely changes the output model by the introduction of a high-resistivity layer at the top and no clear indication of the aquifer — the  $100-\Omega m$  layer. Note that the data are still fitted very nicely (Figure 9b).

A timing error superficially looks similar to a turn-off error as it is set up here. In that sense, a positive timing error would influence the inverted model in a similar way as the turn-off error described in the previous example (i.e., a conductor would be introduced at the top) (Figure 7a).

#### Example 5 — Gate integration

The last example in this class of errors deals with the modeling of the time gates of the Rx. To get a reading from a specific time window the signal is integrated over a gate. This integration can be a simple box-car average or more sophisticated tapered average filters. For narrow averaging kernels the gate value is well represented by the value at the gate-center time. However, especially for late times, some systems have very wide averaging windows and in that case the gate-center time might not be a fair representation of the actual window average. In addition, most systems give the gate-center time as the arithmetic mean of the gate-open and gate-close times. Given that the dB/dt signal on a homogeneous half-space decays proportional to  $t^{(-5/2)}$  (at late times), the geometric mean is a much better representation of the integrated signal than the arithmetic mean. Hence, if the measured integrated signal is modeled using just a gate-center time, the geometric mean of the gate times should be used as the time reference.

To show the effect of the gate integration, we need to modify the nominal system in Table 1. The gate integration error appears for systems with wide gates and especially at late times, to accommodate that, we just delete every second gate of the nominal system. This gives us the gates stated in Table 2.

Using these specifications we can evaluate the effect of gate integration versus a mean value representation. This is shown in Figure 10 for model c in Figure 1. The effect is not very pronounced and for the nominal system with 10 gates per decade the effect is hardly noticeable (not shown).



Figure 4. The effect on the forward response of waveform repetitions on model b. (a) Full decay, (b) a close up of the late times, and (c) a schematic drawing showing the different waveform modeling used. The red curve is one waveform (+; 0) and the green curve is one full cycle (-; 0; +; 0).

However, if we invert the gate-integrated data using a gate-center time approach, we get the results of Figure 11. Using the arithmetic mean as a reference point for the gate value, we get the depth to the deep boundary shifted by 10 m (red curve). Had we used the geometric mean as the reference, the effect on the inverted model would have been very small. Had we used the nominal system of Table 1, the effect on the models would have been very small, even when using the arithmetic mean.



Figure 5. The effect of inverting with only one waveform repetition. The true model is shown with the black line and the inverted model using just one waveform is shown in red. (b) The data fit shown with the same color code as in (a).



Figure 6. The effect on the forward response created by a change in duration of the transmitter turn-off on model a. (a) Full decay and (b) a close up of the early times. The red curve corresponds to a turn-off time of 2  $\mu$ s, the black curve is a turn-off time of 3  $\mu$ s (nominal system), the blue curve is a turn-off time of 4  $\mu$ s, and the green curve is for a 7  $\mu$ s turn-off. (c) The turn-off times shown graphically with the same color-coding.

# Example 6 — Altitude

Until now we have dealt with errors originating from transmitting or receiving instruments. Another class of errors relate to the geometry and positioning of the system. This includes altitude errors, pitch and roll (Tx and Rx together), and for towed-bird systems the individual pitch and roll of the Rx and spatial position of the Rx with reference to the Tx as well.

We will investigate the effect of altitude errors on model b (Figure 1b). The effect on the forward response of varying the height of the frame (Tx and Rx) is shown in Figure 12. As expected, a higher altitude causes a drop in signal level. Generally, changing the altitude affects all of the time-gates, but the early time-gates are most affected.

Again, let us try and invert a data set in which we assume a wrong altitude  $\pm 2$  m. The results are seen in Figure 13. If we do not allow the inversion to change the altitude we see two effects: (1) the depth to the good conductor is moved upward or downward to counteract for the wrong altitude, and (2) the wrong shape of the curves introduced by the wrong altitude is primarily counteracted by changing the thickness of the first layer. Assuming a 20-m altitude when the true altitude is 30 m resembles the canopy effect (orange curves in Figure 13). In this case the model errors are quite significant with the depth to the poor conductor being overestimated by more than 30 m.

However, with the altitudes we have another option — we can invert for the altitude itself by including it as an inversion parameter. By introducing the altitude as an inversion parameter we get, in this case, the true model from the inversion for these noise-free data even if the altitude and the model are not started close to the true model. Resolving the altitude parameter is dependent in general on the conductivity of the top layers. If the top is very resistive there is no contrast to the air and the altitude cannot be resolved, whereas deeper good conductors might still be resolved, but possibly at the wrong depth because of the wrong altitude.

#### **Example 7** — Combined effects

Finally, we will try to simulate real-life situations in which more than one effect plays a role at the same time. In Figure 14 the effect of variations in the specifications of the system transfer function on the forward response of model b in Figure 1 are shown. We have modeled the combined effects of an incorrect low-pass filter estimation, modeling only one waveform repetition, a slightly wrong altitude, and disregarding the gate integration. From Figure 14 it is clear that the low-pass filter effect dominates at early times where it suppresses the wrong altitudes. At late times the effect of the waveform repetition is by far the dominating feature. The effect of the gate integration cannot be seen because of the narrow gates in the nominal system used here.

Again, let us try and invert the nominal data set with these offsets in the system transfer function. To resemble real conditions we also invert for the altitude this time. The distinctive results of the wrong assumptions are shown in Figure 15. Overestimating the low-pass filter frequency affects the early times and introduces a significant underestimation of the thickness of the first layer (blue line). The missing waveform repetition affects the late times and shifts the good conductor downward (blue and red lines). However, note that the altitudes are also shifted significantly from the true value. Because variation in the altitude has a strong effect on all gates, the altitude is the parameter that tends to be changed when there is a



Figure 7. The effect on the inverted model assuming a  $3-\mu s$  turn-off time when it is actually 4 or 7  $\mu s$ . (a) The inverted models assuming a  $3-\mu s$  turn-off are shown with blue ( $4-\mu s$  true) and red ( $7-\mu s$  true); the true model is shown in black. (b) The near-perfect data fits.



Figure 8. The effect on the forward response of Tx-Rx timing errors on model a. The black curve shows no timing error (nominal system), the red curve is an error of  $-6 \ \mu$ s, the blue curve is an error of  $-3 \ \mu$ s, the green curve is an error of  $+3 \ \mu$ s, and the purple curve is an error of  $+6 \ \mu$ s. Positive values mean that the current pulse is closer to the gates; negative values mean that the current pulse is further from the gates.

modeling error that cannot be accounted for with one of the other parameters that is being inverted for. The data fits in Figure 15b are acceptable, which means that these results would most likely be accepted and the altitude shift would be regarded as a canopy effect, pitch and roll of the system, or errors associated with the altitude processing. If we did not incorrectly model the filters and waveform repetition, the altitude would have been inverted to the correct 30 m without any problems. Leaving out the altitude in the inversion



Figure 9. The effect on the inverted model having a timing error on data from model a. (a) The inverted model with a timing error of 3  $\mu$ s is shown with blue and red is a timing error of 6  $\mu$ s. The true model is black. (b) The data fits shown with the same color code.

Table 2. Gate times in seconds for a wide-window system

Open (s)	Close (s)	Arithmetic center (s)	Geometric center (s)
$9.741 \times 10^{-6}$	$1.544 \times 10^{-5}$	$1.259 \times 10^{-5}$	$1.226 \times 10^{-5}$
$1.544 \times 10^{-5}$	$2.447 \times 10^{-5}$	$1.995 \times 10^{-5}$	$1.943 \times 10^{-5}$
$2.447 \times 10^{-5}$	$3.878 \times 10^{-5}$	$3.162 \times 10^{-5}$	$3.080 \times 10^{-5}$
$3.878 \times 10^{-5}$	$6.146 \times 10^{-5}$	$5.012 \times 10^{-5}$	$4.882 \times 10^{-5}$
$6.146 \times 10^{-5}$	$9.741 \times 10^{-5}$	$7.943 \times 10^{-5}$	$7.737 \times 10^{-5}$
$9.741 \times 10^{-5}$	$1.544 \times 10^{-4}$	$1.259 \times 10^{-4}$	$1.226 \times 10^{-4}$
$1.544 \times 10^{-4}$	$2.447 \times 10^{-4}$	$1.995 \times 10^{-4}$	$1.944 \times 10^{-4}$
$2.447 \times 10^{-4}$	$3.878 \times 10^{-4}$	$3.162 \times 10^{-4}$	$3.080 \times 10^{-4}$
$3.878 \times 10^{-4}$	$6.146 \times 10^{-4}$	$5.012 \times 10^{-4}$	$4.882 \times 10^{-4}$
$6.146 \times 10^{-4}$	$9.741 \times 10^{-4}$	$7.943 \times 10^{-4}$	$7.737 \times 10^{-4}$
$9.741 \times 10^{-4}$	$1.544 \times 10^{-3}$	$1.259 \times 10^{-3}$	$1.226 \times 10^{-3}$
$1.544 \times 10^{-3}$	$2.447 \times 10^{-3}$	$1.995 \times 10^{-3}$	$1.943 \times 10^{-3}$
$2.447 \times 10^{-3}$	$3.878 \times 10^{-3}$	$3.162 \times 10^{-3}$	$3.080 \times 10^{-3}$
$3.878 \times 10^{-3}$	$6.146 \times 10^{-3}$	$5.012 \times 10^{-3}$	$4.882 \times 10^{-3}$
$6.146 \times 10^{-3}$	$9.741 \times 10^{-3}$	$7.943 \times 10^{-3}$	$7.737 \times 10^{-3}$

should not be an option because all canopy effects would migrate directly to the model.

This example illustrates the difficulties associated with modeling and inversion of AEM data. If the system transfer function is not modeled accurately, the errors introduced may migrate to the model parameters — earth or geometry related. This points to the fact that not only is it important to model the system transfer function correct-



Figure 10. The effect on the forward response of modeling the response of model c with just the gate-center time (red curve) or with actual signal integration over the gate width (black curve). (a) Full decay and (b) a close up of the late times.



Figure 11. (a) The effect on the inverted model by modeling with a gate-center time at the arithmetic mean (red curve) or the geometric mean (blue curve). The true model is black. (b) The data fits shown with the same color code.

ly (obviously!), but it is also important to always analyze the output carefully (e.g., for systematic errors that might point to wrong filter settings or indications of other issues as discussed in this section).

Including altitude in the inversion calls for special attention. First, it is mandatory to feed it with a feasible a priori value, typically ob-



Figure 12. The effect on the forward response of altitude differences on model b. The black curve shows the nominal altitude at 30 m, the orange curve is 20 m, the red curve is 26 m, the blue curve is 28 m, the green curve is 32 m, and the purple curve is 34 m.



Figure 13. The effect on the inverted model having an assumed altitude error of 2 m on data from model b. (a) The inverted model assuming 28-m altitude (true = 30 m) is shown with green, assuming 32 m is blue, and assuming 20 m (canopy) is orange. The true model is black. By including the altitude in the inversion, we get exactly the true model (behind the black line). (b) The almost-perfect data fits.

tained from carefully processed data from the laser altimeters. Then it is crucial to constrain it along the flight path to resemble the recording conditions of slowly varying altitudes. Finally, the altitude should only be allowed to absorb the part of the data misfit that is not easily described by resistivity structures.

#### DISCUSSION

# Errors for actual systems

In the paragraphs above we have dealt only with a nominal system. Obviously, not every effect affects the different systems in the same way because of the individual system setups and data handling and processing. One should also bear in mind that the severity of these effects is model dependent. The summarizing comments below are based on our experiences with different systems. We will avoid mentioning specific names and leave it to the reader to ask their data supplier for specifications. Furthermore, many of the systems have had several generations (summarized nicely by Allard, 2007), and in some cases new and old generations are still in operation. Our experiences are with data from systems flow in the last couple of years, which should reflect the latest generations of hardware and software.

Filter effects are mostly seen with systems that use filters with very low cut-off frequencies and high order. We have seen filters with cut-off frequencies as low as third-order 5 kHz. The fact that sometimes operating companies do not disclose filter information (because it is considered proprietary information) worsens the effect and significantly influences the interpretation of those data. As demonstrated, it is necessary to know to what degree the primary field



Figure 14. The effect on the forward response of several modeling errors on model b. The black curve shows the response from the nominal altitude at 30 m with 70-kHz filtering, two waveform repetition, and gate integration. The red and blue curves show a combination of effects. They are both modeled at an altitude of 28 m using only one waveform repetition and no gate integration. The blue curve has a low-pass filter of 75 kHz and the red at 65 kHz.

was compensated, primarily for systems that use filters lower than a few tens of kilohertz. Butterworth filter characteristics have been assumed, but because other low-pass filters are in use, their details are important for quantitative modeling and need to be provided by contractors.

Waveform repetition is only a modeling issue; the information is widely available. If ignored, models from all systems are affected. However, it is most severe at late time measurements, therefore pointing at the large-moment, low-base frequency systems.

Correct modeling of turn-off time affects data from systems with a long turn-off time in combination with high moments. The problem is not the modeling itself but the fact that such systems seem to have a slight drift in the actual shape of the turn-off ramp. System drift and leveling issues makes the quantification of possible errors with turnoff times even more difficult.

Timing issues are mostly a problem when combining early time measurements with high moments, which is very desirable to get near-surface and deep information. This kind of error is very hard to quantify; however, in our experience, many systems have this problem.

Gate-integration is also a pure modeling issue: the longer the gates, the more severe the distortion. If ignored, all systems are affected.

Altitude information is available from all systems. This is the nominal operating altitude, a GPS altitude of the helicopter, or the actual frame altitude obtained with laser altimeters of centimeter accuracy. To our knowledge, only one helicopter-based TEM system systematically records the pitch and roll of the frame, allowing for accurate vertical corrections of the true frame height and frame area. Canopy effect also frequently plays a role for all systems but can most often be dealt with by advanced filtering. Because of these dif-



Figure 15. The effect on the inverted model having a combination of modeling errors on data from model b. (a) The inverted model assuming 28-m altitude (true = 30 m), one waveform repetition, no gate integration, and a 65-kHz low-pass filter is shown in red. Assuming all of the same except for a 65-kHz filter is shown in blue. The true model is black. Note that the altitude is also included in the inversion and is shown on top of the models. (b) The decent data fits.

ferent degrees of inaccuracies in the input altitude, a proper modeling must incorporate all available information and solve for altitude during the inversion. This is obviously more crucial for systems that only provide nominal altitude or maybe altitudes not measured from the Tx frame but from the helicopter.

#### **Fixed-wing systems**

This paper has dealt with a nominal, central-loop airborne TEM system measuring only the z-component of the field. Thus, the nominal setup applies mainly to the many helicopter systems in operation. There is another class of systems often referred to as fixed-wing systems. They have the Tx strung around a traditional aircraft with the receivers located in a bird towed behind. Normally, the x, y, and z-components are measured. In general, the conclusions drawn above also apply to a fixed-wing system. However, a fixed-wing system has some additional issues that are not included in the analyses performed here. In the fixed-wing system we have a bird moving in the airspace independent of the Tx. The movements of the bird are normally not recorded, but it is well known that there are periodic movements of the bird (Smith and Annan, 1997; Smith, 2001; Davis et al., 2006; 2009). These movements introduce pitch and roll to the Rx separately from the Tx. The pitch of the Rx has an especially large influence on the fields received. It has been shown that it is important to include at least the pitch in the forward modeling (Brodie and Sambridge, 2006; Auken et al., 2009), but even better to actually include it as an inversion parameter similar to what was previously described with the altitude in this paper. Similarly, there can be significant errors associated with the inductive effect of the aircraft itself, although very little information is available on this issue.

#### **Calibration issues**

In all of the examples presented here we assumed that no calibration or bias-issues were present in the data. Obviously, if this is the case on top of any of the issues presented in this paper, the conclusions drawn potentially get worse.

#### CONCLUSIONS

For airborne TEM, errors are introduced in the models if the system specifications used in the forward modeling are inexact. The inexact description can come from insufficient specifications from the operating company and/or from the modeling software being unable to actually handle that information.

Using a nominal airborne TEM system on three different models we have quantified the effect of even small errors associated with filters, Tx waveform, timing between Tx and Rx, integration over gates, altitude errors, and combined effects.

In many of the cases we see layer boundaries shift more than 10 m for errors that are likely with real data. In some cases resistivities of layers also change quite dramatically.

Needless to say, for high-accuracy modeling of, for example, an

aquifer system, the model errors reported here may lead to serious misconceptions. To avoid this, accurate system description is needed on the operator side and on the modeling side.

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